Differentiation Formulae

Eg: $f(x) = \sqrt{x} = x^{1/2}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{\cancel{x} + h - \cancel{x}}{h\left(\sqrt{x+h} + \sqrt{x}\right)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h} + \sqrt{x}\right)} \qquad x \geq 0$$

$$x \neq 0$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}}$$

$$\frac{d}{dx} x^{1/2} = \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2} x^{-1/2}$$

$$n x^{n-1}$$

$$x^{1/2} \qquad \frac{1}{2} x^{1/2 - 1}$$

$$\boxed{\begin{array}{ll} \sqrt{2x+1} & \dfrac{1}{3x+2} \\ x^2 - x & \end{array}}$$

no trig

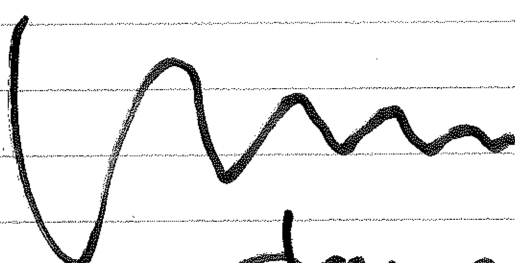
$nx^{n-1} = f'(x)$

$f(x) = x^n$

for any n!

Product Rule

Eg: $\frac{1}{x}\sin x$



damped oscillation

Eg: $f(x) = x^2 e^x$

$$\lim_{h \to 0} \frac{(x+h)^2 e^{x+h} - x^2 e^x}{h}$$

$$- x^2 e^{x+h} + x^2 e^{x+h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 e^{x+h} \underbrace{- x^2 e^{x+h} + x^2 e^{x+h}} - x^2 e^x}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 e^{x+h} - x^2 e^{x+h}}{h} + \frac{x^2 e^{x+h} - x^2 e^x}{h}$$

$$\lim_{h \to 0} e^{x+h} \cdot \overbrace{\left[\frac{(x+h)^2 - x^2}{h} \right]}^{2x}$$

$$+ \lim_{h \to 0} x^2 \underbrace{\frac{e^{x+h} - e^x}{h}}_{e^x}$$

$$= e^x \cdot 2x + x^2 e^x$$

$$= 2x e^x + x^2 e^x$$

$$\frac{d}{dx} f(x) \cdot g(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

We'll see

$$\frac{d}{dx}\sin x = \cos x$$

$$g(x) = \frac{1}{x}\sin x$$

$$= x^{-1}\sin x$$

$$g'(x) = -1 \cdot x^{-2}\sin x + x^{-1}\cos x$$

$$= \frac{1}{x}\cos x - \frac{1}{x^2}\sin x$$

$$(u+v)' \quad \boxed{(uv)' = u'v + uv'}$$

Eg: $u = \dfrac{1+2x}{\sqrt{x}}$

$$\frac{du}{dx}$$

$$u = \frac{1}{\sqrt{x}} + \frac{2x}{\sqrt{x}}$$

$$= x^{-1/2} + 2x^{1/2}$$

$$\frac{du}{dx} = -\tfrac{1}{2}x^{-3/2} + 2 \cdot \tfrac{1}{2}x^{-1/2}$$

$$(uv)' = u'v + uv'$$

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

Ex: $\overset{f(x)}{\cancel{y}} = \dfrac{\sqrt{x}}{1+2x}$

$$= \sqrt{x} \cdot \frac{1}{1+2x}$$

$$\left(\frac{u}{v}\right)' = \frac{v u' - u v'}{v^2}$$

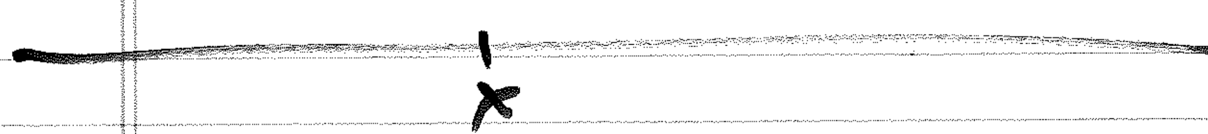
careful

$$f'(x) = \frac{(1+2x) \cdot \frac{1}{2\sqrt{x}} - \sqrt{x} \cdot 2}{(1+2x)^2}$$

Eg: $x = t^4 e^t$

feet

seconds



$v = \frac{dx}{dt}$ $\frac{ft}{sec}$

$= 4t^3 e^t + t^4 e^t$

$= (4t^3 + t^4) e^t$

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

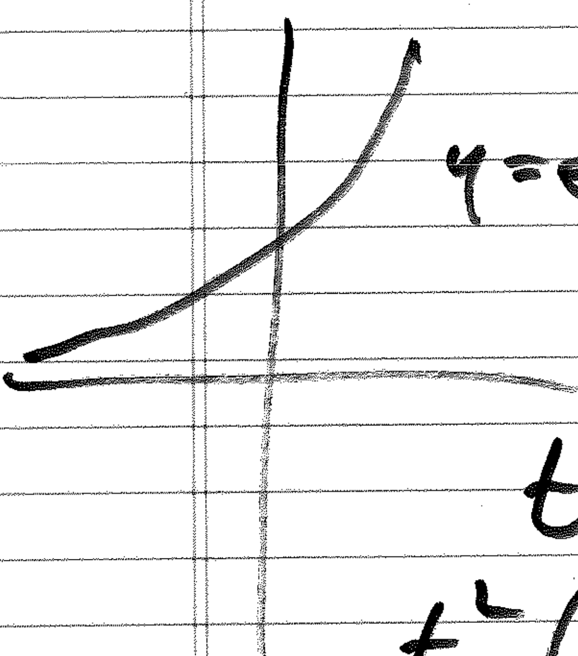
$\frac{ft/sec}{sec}$

$t = -6, -2, 0$

$$a = (12t^2 + 4t^3)e^t + (4t^3 + t^4)e^t$$

$$a = 0 \;?$$

$$(12t^2 + 8t^3 + t^4)\,\cancel{e^t} = 0$$



$$t^4 + 8t^3 + 12t^2 = 0$$

$$t^2(t^2 + 8t + 12) = 0$$

$$t^2(t+2)(t+6) = 0$$

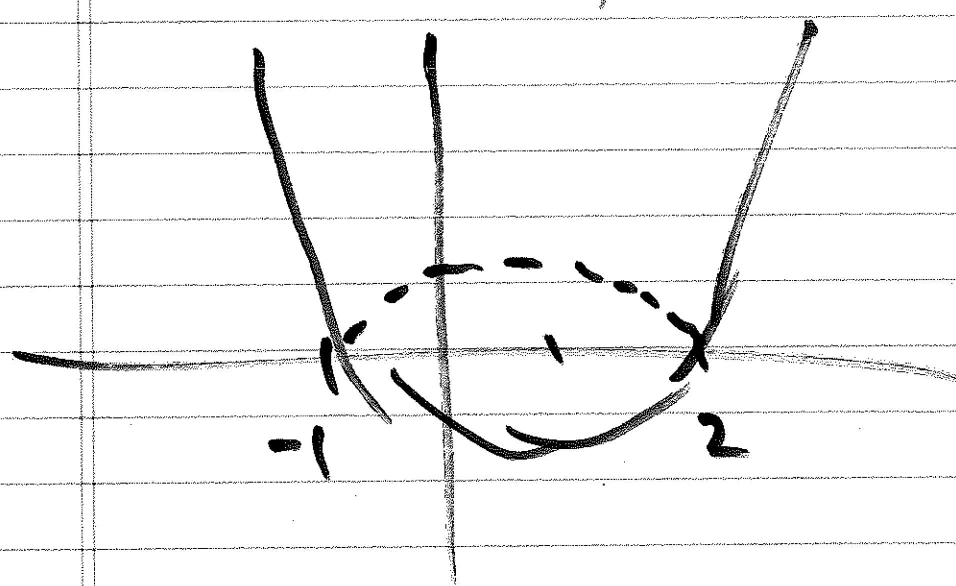
Eg: ~~y=|t~~

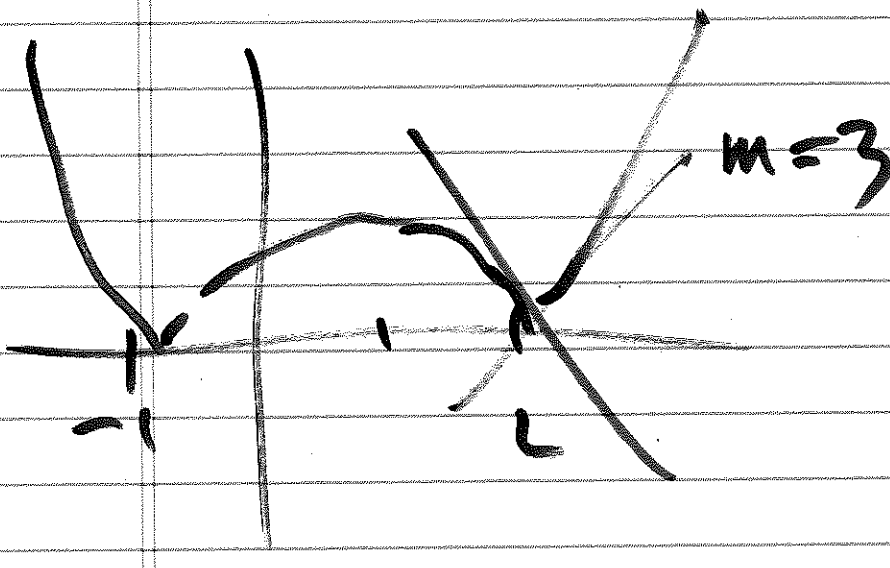
$$y = |t^2 - t - 2|$$

Sketch

$$y = t^2 - t - 2$$

$$(t-2)(t+1)$$





$$y = \begin{cases} t^2-t-2 & \text{otherwise} \\ -(t^2-t-2) & -1 \le t \le 2 \end{cases}$$

$$\frac{dy}{dt} = \begin{cases} 2t-1 & t<-1 \text{ or } t>2 \\ -2t+1 & -1<t<2 \end{cases} \checkmark$$

$$\left.\frac{dy}{dt}\right|_{t=3/2} = -2(3/2)+1 = -2$$

where is the tangent $\perp$ to $2y + x = 1$?

$y = -\frac{1}{2}x + 1/2$

$m_{tan} = 2$ $\qquad$ $m = -1/2$

① $2t - 1 = 2$

$2t = 3$

$t = 3/2$ $\qquad$ $3/2 < -1$!

$3/2 > 2$?

② $-2t + 1 = 2$ ✓

$-2t = 1$

$t = -1/2$ $\qquad$ $-1 < -\frac{1}{2} < 2$

✓