Derivative

$$\frac{d}{dx} x^5$$

$$\lim_{h \to 0} \frac{(x+h)^5 - x^5}{h}$$

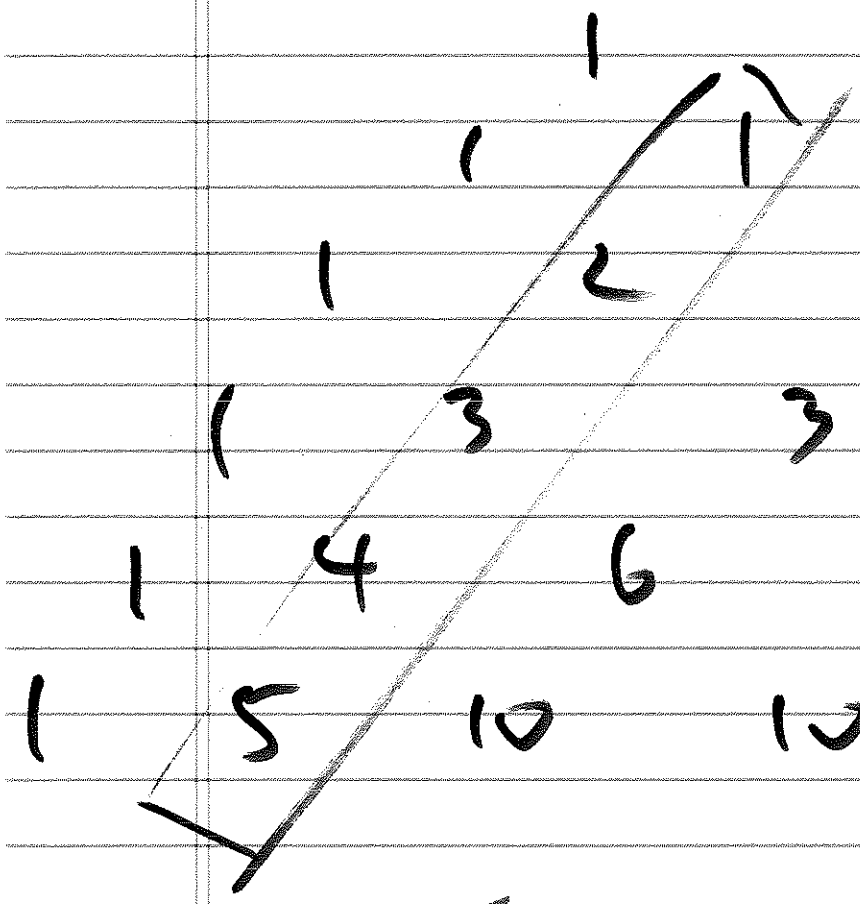
Scratch

$$(x+h)^5 =$$

$$x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5$$

$$x^5 + 5x^4h + h^2(\text{junk})$$

Pascal's Triangle



$$\lim_{h \to 0} \frac{x^5 + 5x^4 h + h^2(\quad) - x^5}{h}$$

$$= \lim_{h \to 0} \frac{5x^4 h + h^2(\quad)}{h}$$

$$= \lim_{h \to 0} \frac{k \cdot (5x^4 + h(\text{junk}))}{k}$$

$$= \lim_{h \to 0} 5x^4 + h(\text{junk})$$

$$= 5x^4$$

Thm: $\frac{d}{dx} x^n = n x^{n-1}$

Thm: $\frac{d}{dx}(f(x) + g(x))$

① $= f'(x) + g'(x)$

② $\frac{d}{dx}(a \cdot f(x)) = a \cdot \frac{d}{dx} f(x)$

Ex: $\frac{d}{dx} x^4 + 3x - 2$

$= \frac{d}{dx} x^4 + \frac{d}{dx} 3x - \frac{d}{dx} 2$

$= 4x^3 + 3\frac{d}{dx} x - 2\frac{d}{dx} 1$

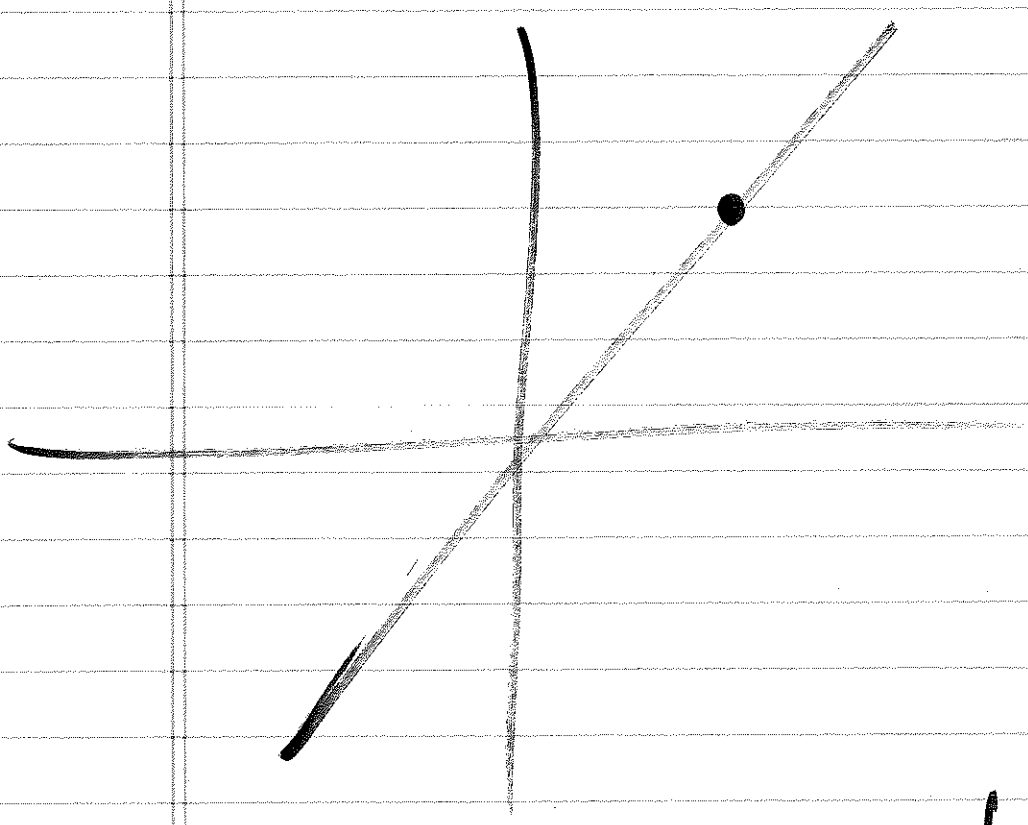
$1 \cdot x^0$ $\quad$ x^0

$0 \cdot x^{-1}$

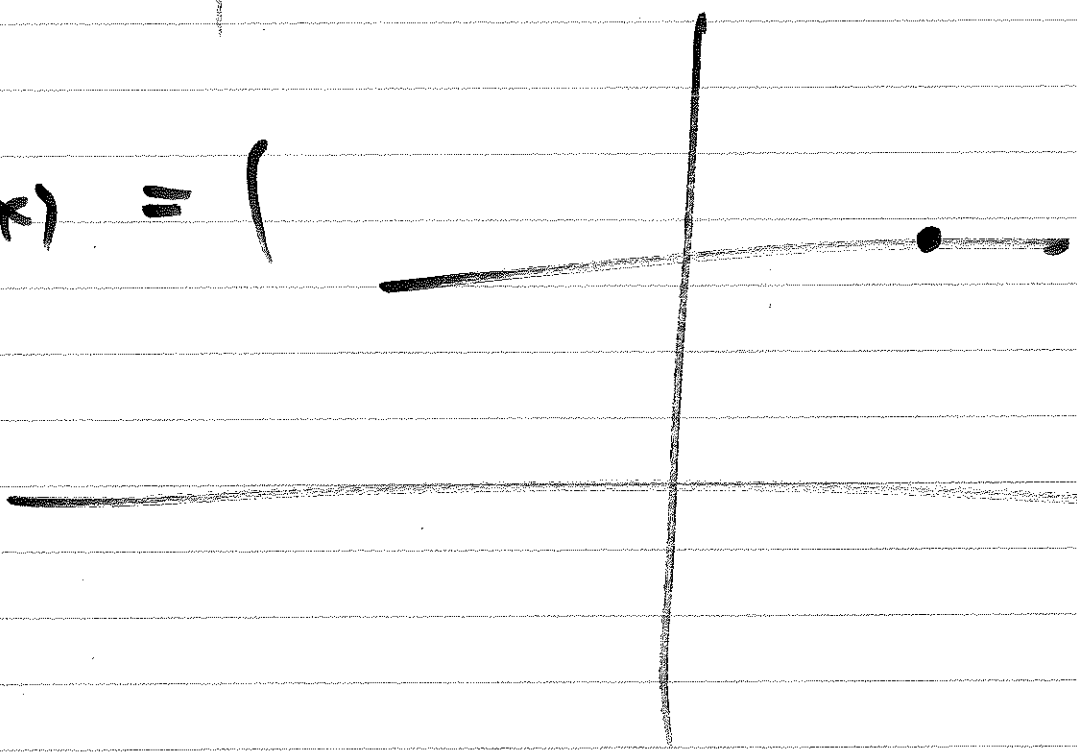
$= 4x^3 + 3 \cdot 1 - 2 \cdot 0$

$= 4x^3 + 3$

$f(x) = x$



$f(x) = 1$



The Exponential Function

$f(x) = e^x$

$e \approx 2.7$

$$\lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= \lim_{h \to 0} \frac{e^x \cdot (e^h - 1)}{h}$$

$$= e^x \cdot \lim_{h \to 0} \frac{e^h - 1}{h}$$

$$= e^x \cdot 1 \qquad \text{table}$$

$$\frac{d}{dx} e^x = e^x$$

$f'(x) = f(x)$ Diff Eq

Eg: $y = x^2 + 5e^x$

$$\frac{dy}{dx} = 2x + 5e^x$$

$$\frac{d^2y}{dx^2} = 2 + 5e^x$$

2nd derivative

—y—

Ex: $y = x^2 + 3$

Find (a, b) on the parabola such that the tangent at (a, b) passes through $(1, 0)$

$y - b = m(x - a)$

$b = a^2 + 3$

$m = \frac{dy}{dx} = 2x$

$m = 2a$

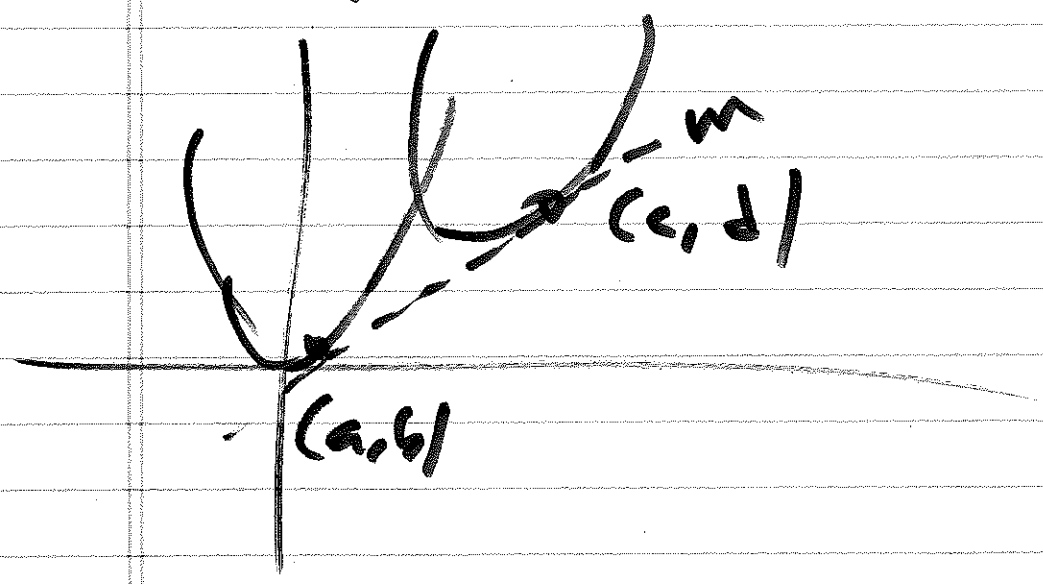
$a^2 - 2a - 3 = 0$

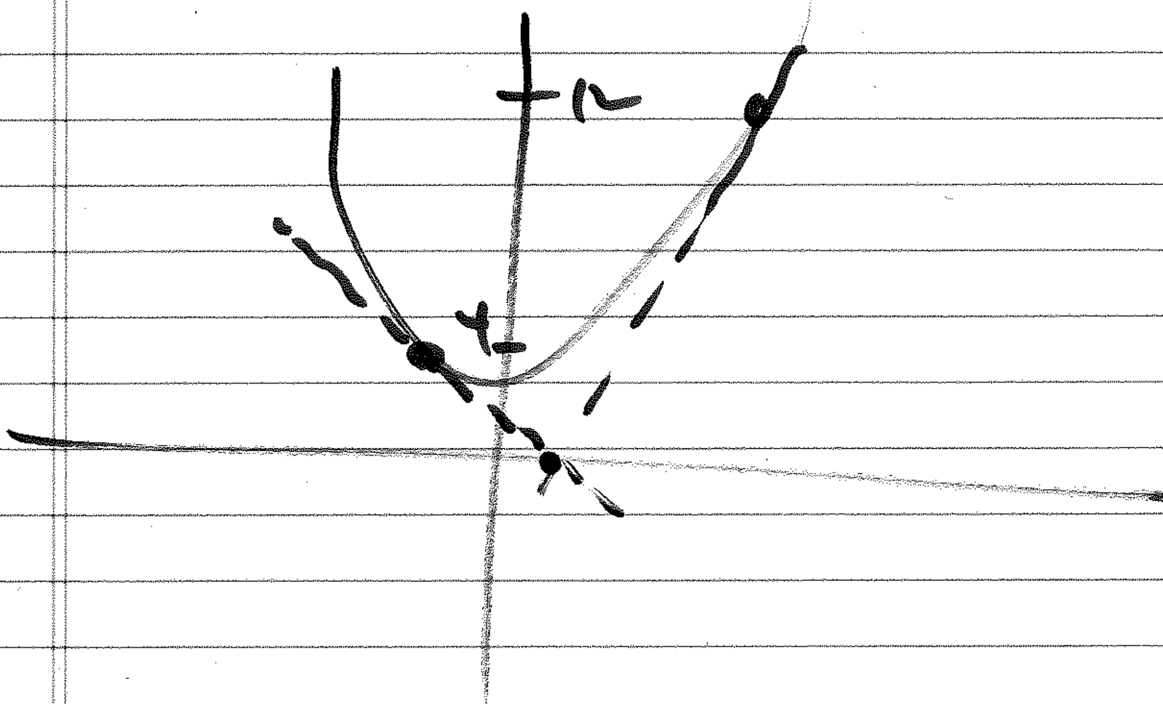
$(a+1)(a-3) = 0$

$a = -1, 3$

$(-1, 4)$ $(3, 12)$

Bitangents





$$y - (a^2 + 3) = 2a(x - a)$$

($y = 0$, $x = 1$)

$$-a^2 - 3 = 2a(1 - a)$$

$$= 2a - 2a^2$$

$$a^2 - 3 = 2a$$

$$-a^2 + 3 = 2a - 2a^2$$

$y = x^2$ $\qquad \frac{dy}{dx} = 2$

$y = (x-1)^2 + 1 \quad = x^2 - 2x + 2$

$\frac{dy}{dx} = 2x - 2$

$b = a^2$

$d = c^2 - 2c + 2$

$m = \frac{d-b}{c-a}$

$= 2a$

$= 2c - 2$

$2a = 2c - 2$

$a = c - 1$

$2a = \frac{d-b}{c-a}$