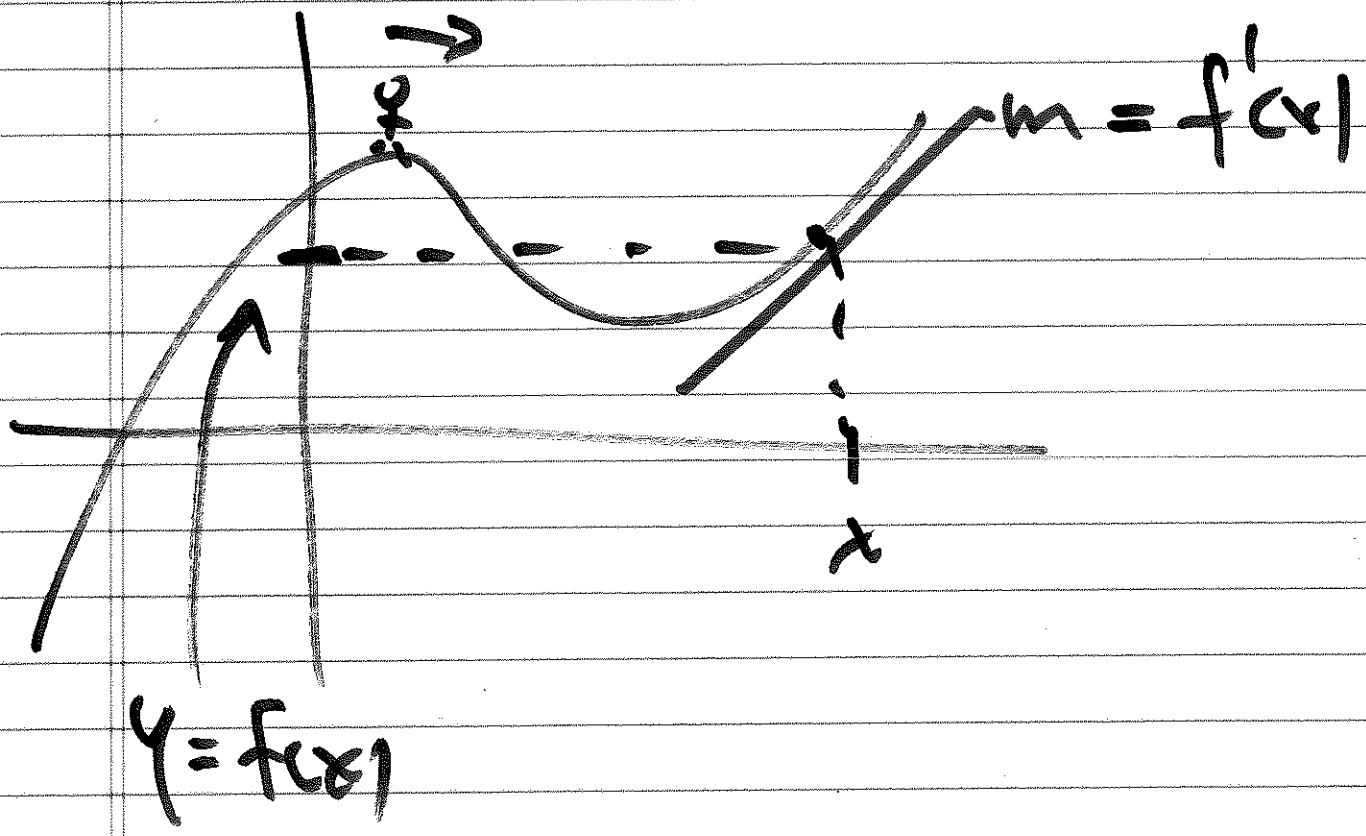
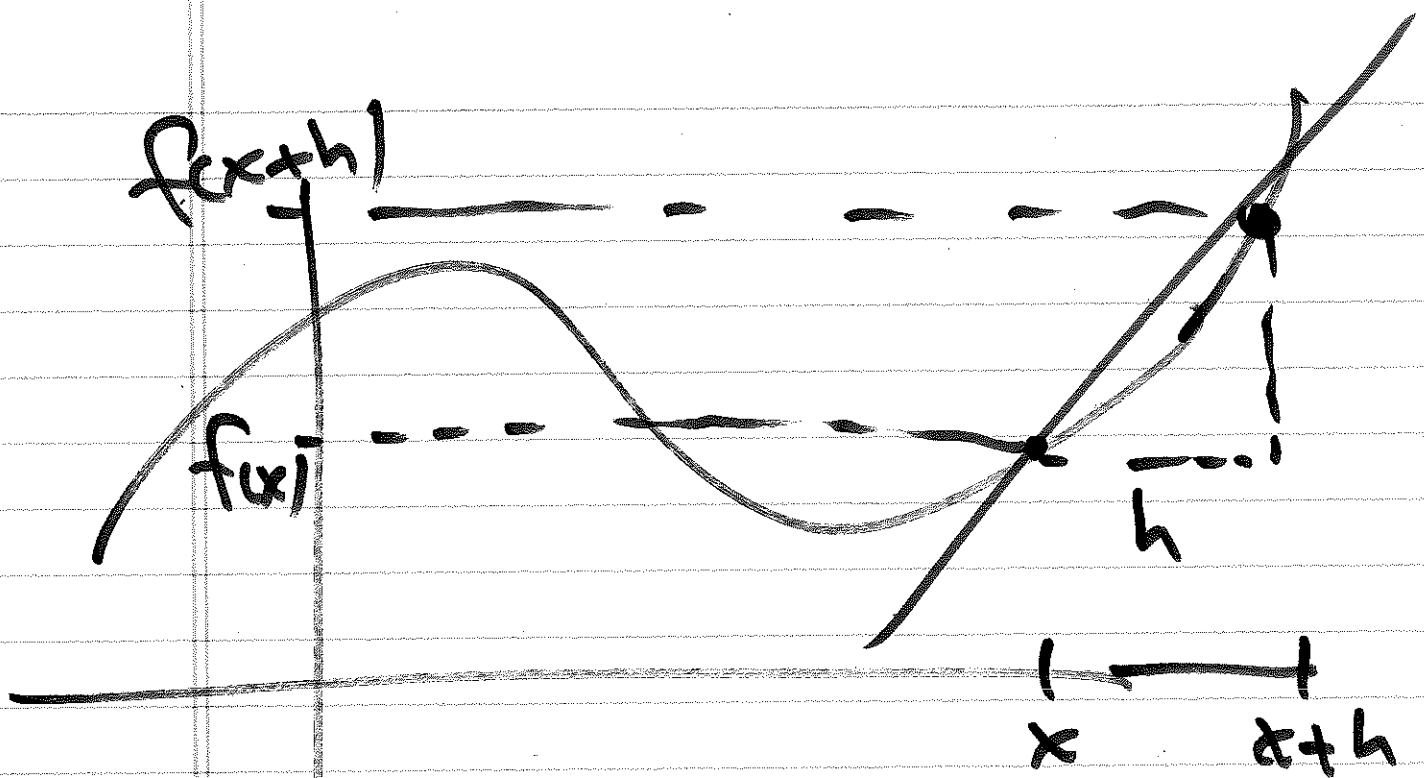
Derivative

$y = f(x)$

$f'(x)$ $\quad \frac{dy}{dx}$ $\quad \frac{\Delta y}{\Delta x}$





$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

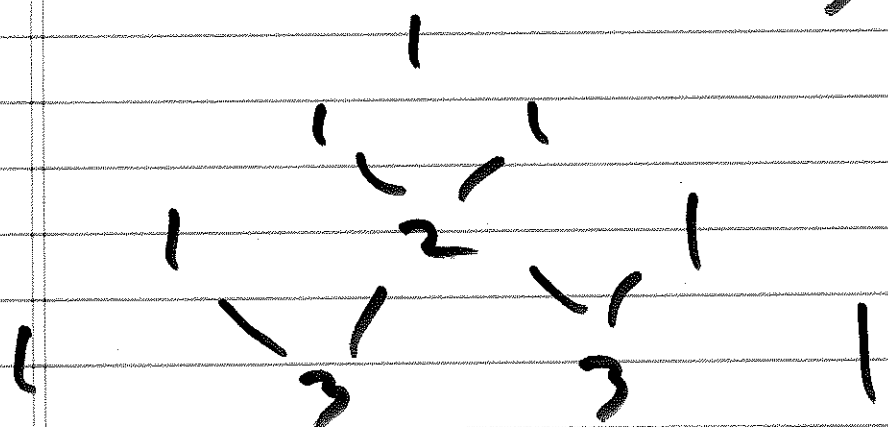
Eg. $f(x) = x^3$

$$f'(a) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

scratch

$(x+h)^3 = ?$

Pascal's triangle



$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

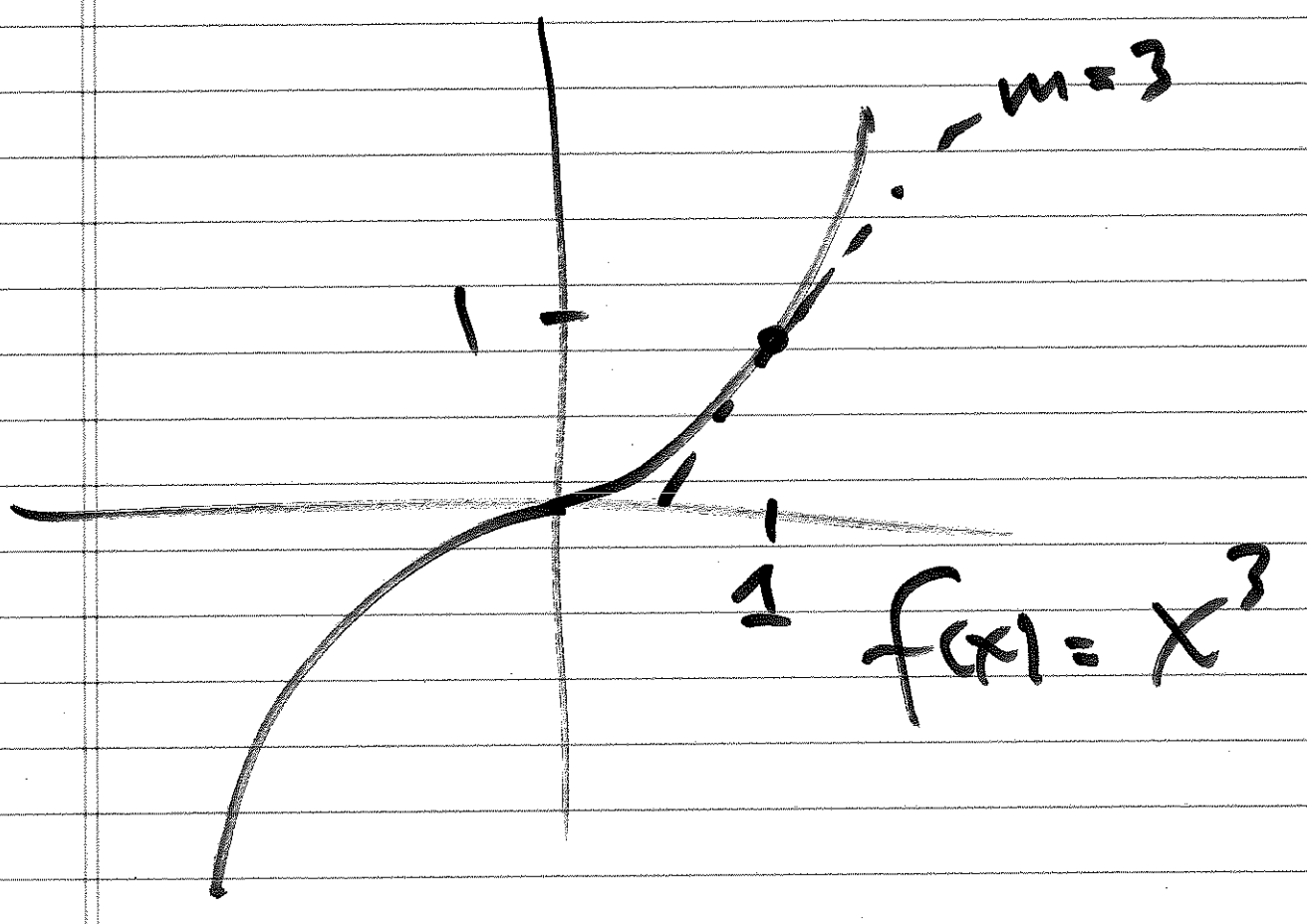
$$(x+h)^3 - x^3 = 3x^2h + 3xh^2 + h^3$$

$$f'(x) = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$= \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2)}{h}$$

($3xh \to 0$, $h^2 \to 0$)

$$= 3x^2$$

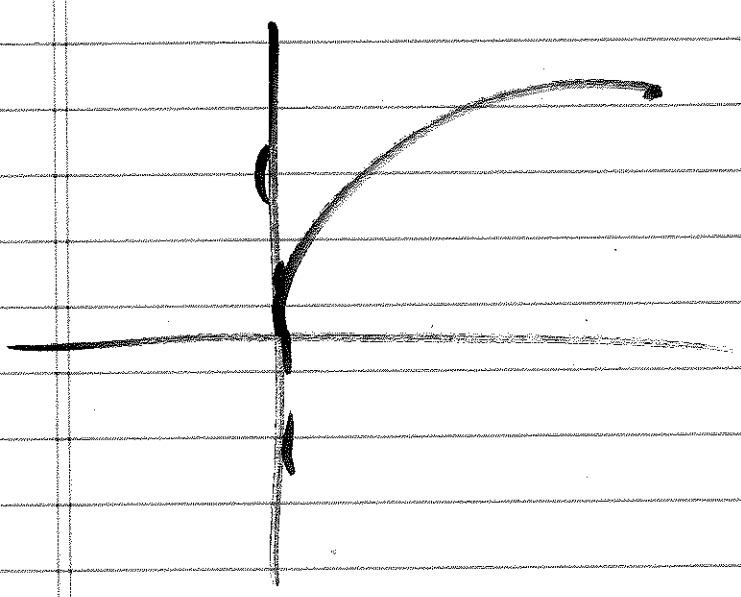

m=3
1
1
f(x) = x^3

$f'(x)$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

eg. $f(x) = \sqrt[3]{x}$

doesn't converge if $x = 0$



differentiable fcn

$\frac{0}{0}$ $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$

If f is diff'ble

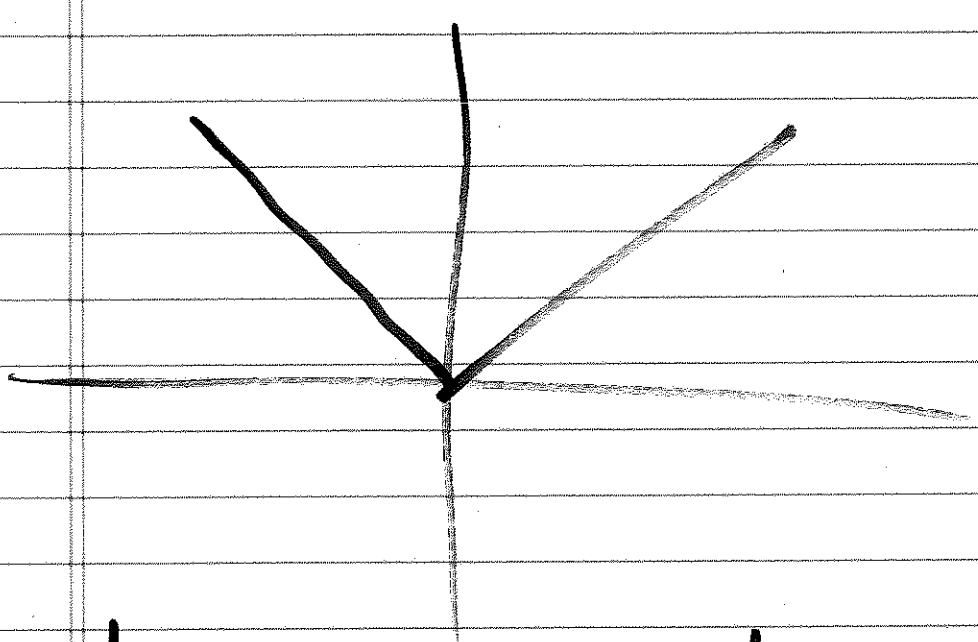
then $\lim_{x \to a} f(x) - f(a) = 0$

$\therefore \lim_{x \to a} f(x) = f(a)$

f is cont at a

Ex: $g(x) = |x|$

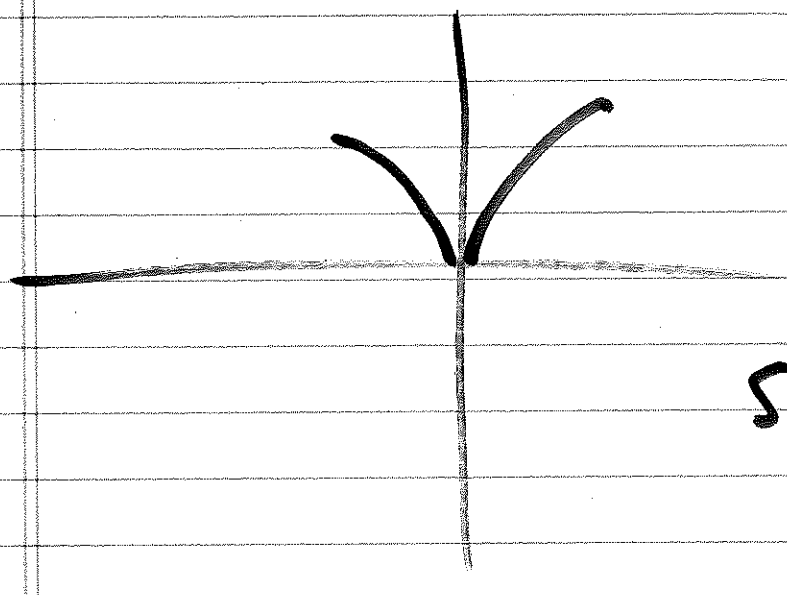
$$= \begin{cases} x & \text{if } x \geq 0 \\ -x & x < 0 \end{cases}$$



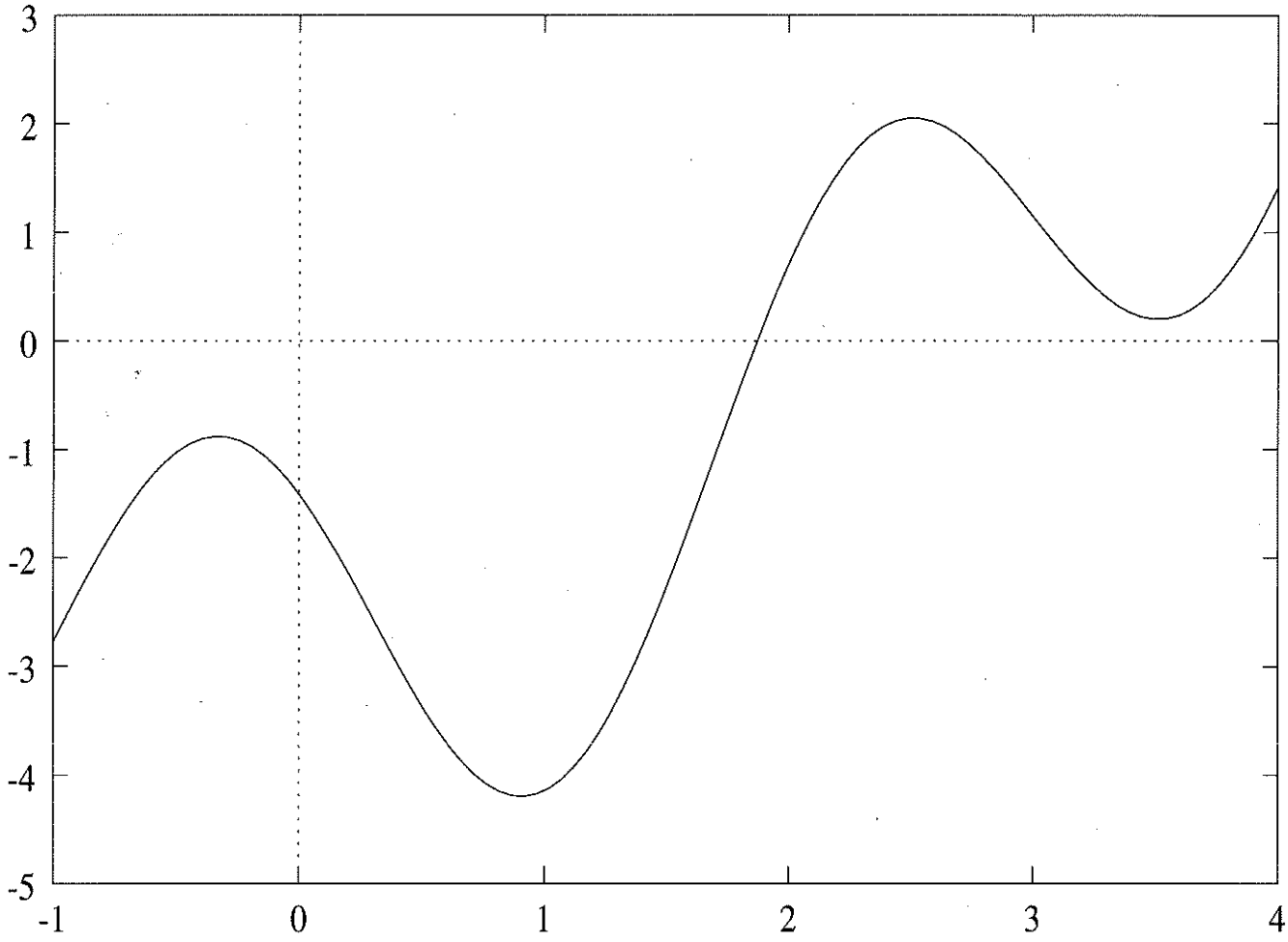
$$\lim_{x \to 0^+} \frac{g(x) - g(0)}{x - 0} = 1$$

$$\lim_{x \to 0^-} \quad = -1$$

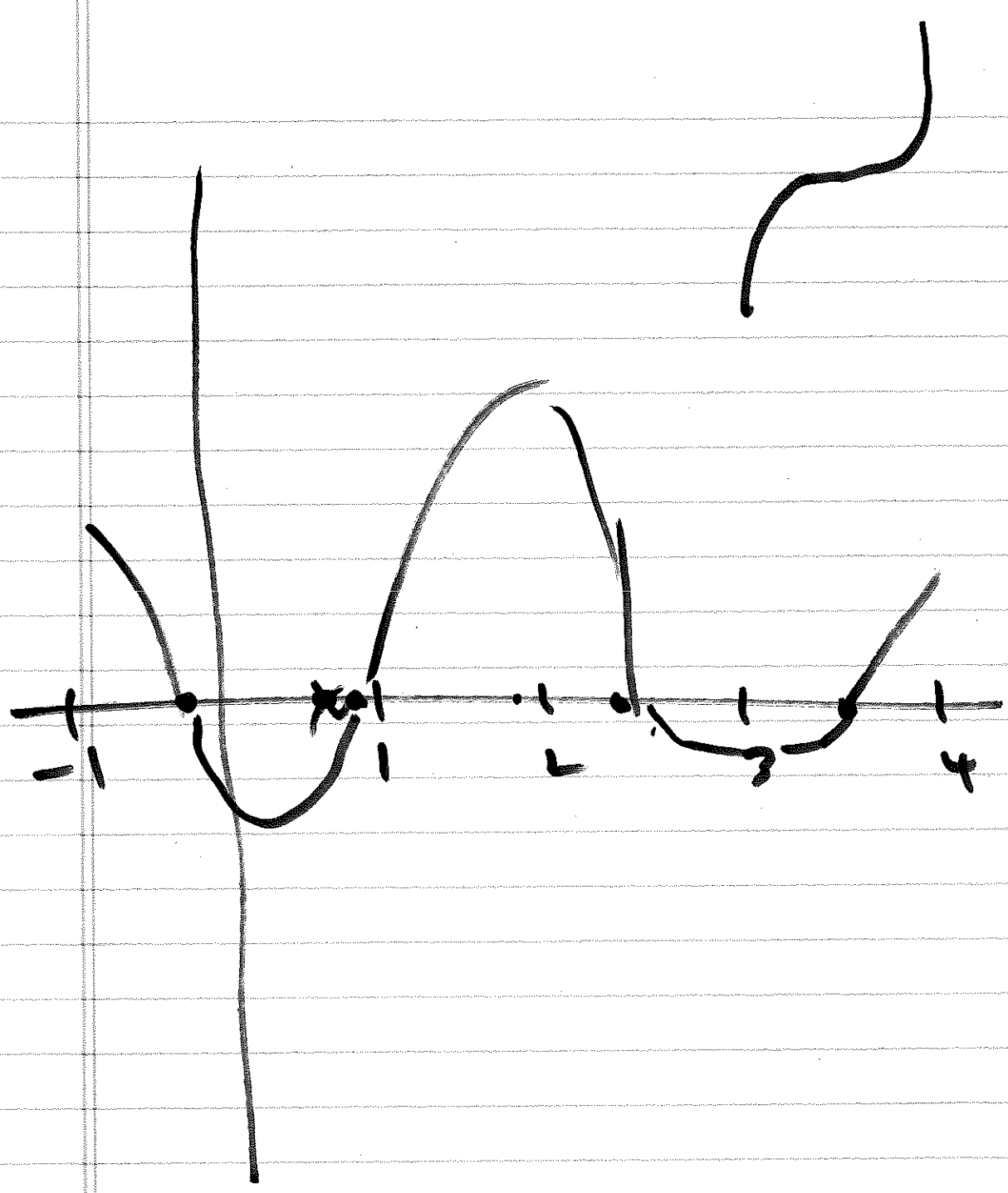
$f(x) = x^{4/3}$



smooth



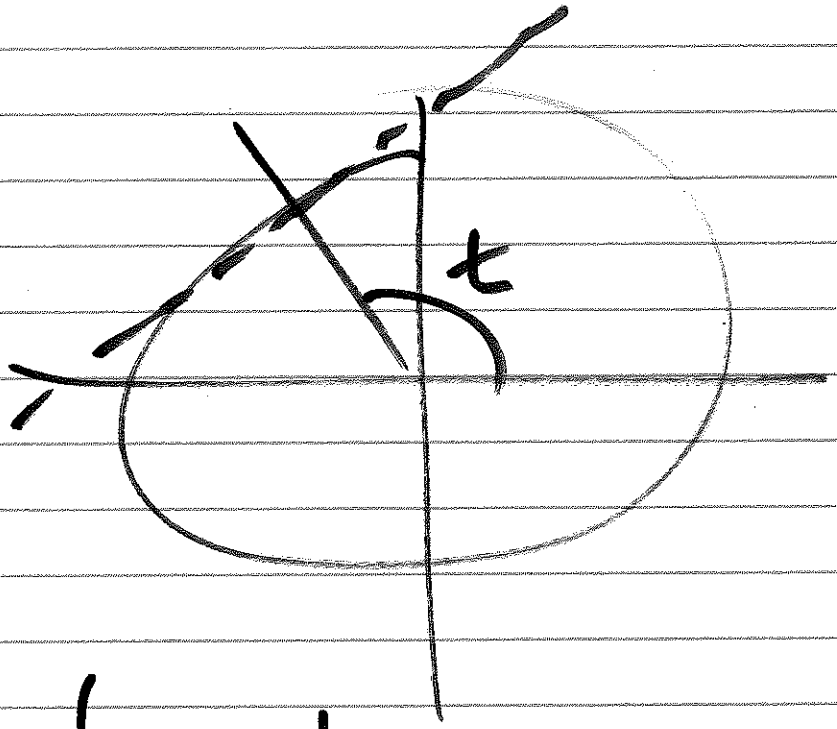
Sketch the Derivative



pos/neg

zero

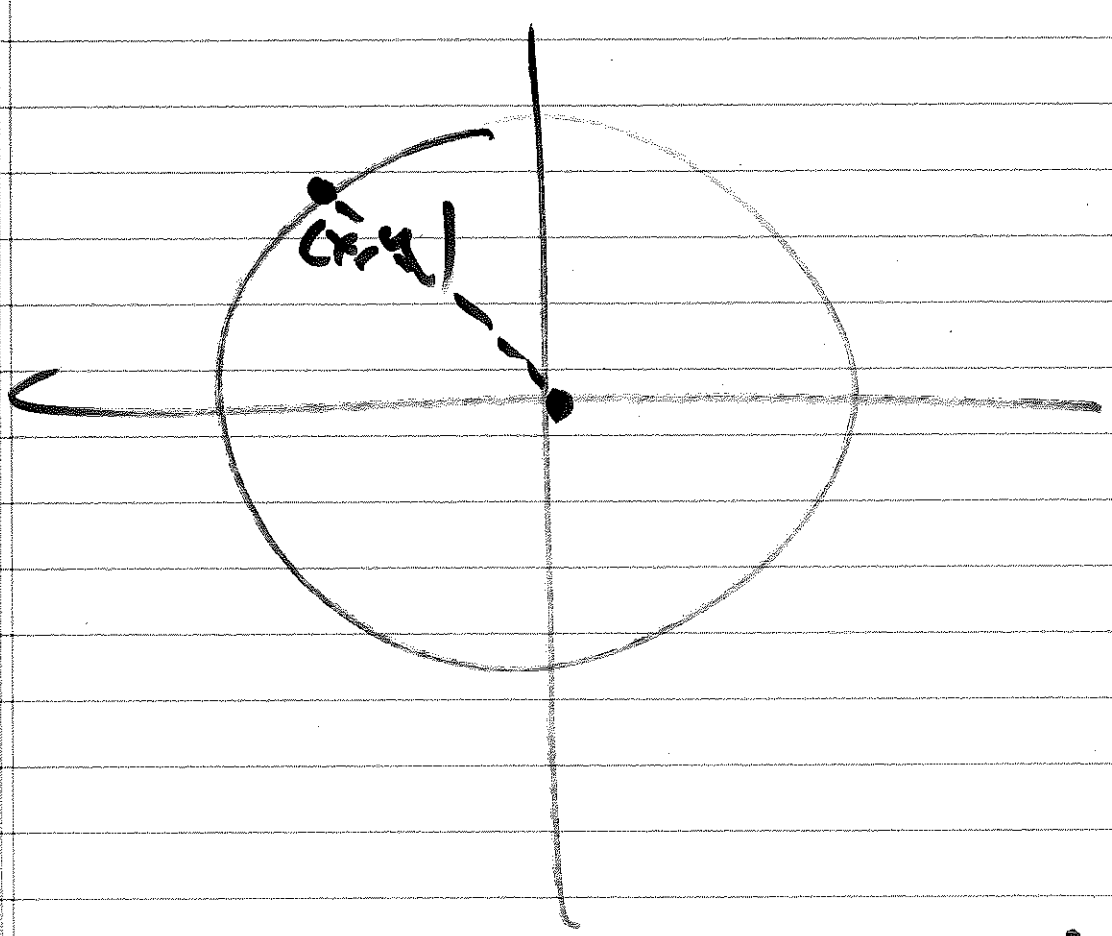
m=0



tan line

parametric equations for circle

$$\begin{cases} x = \cos t \\ y = \sin t \end{cases}$$

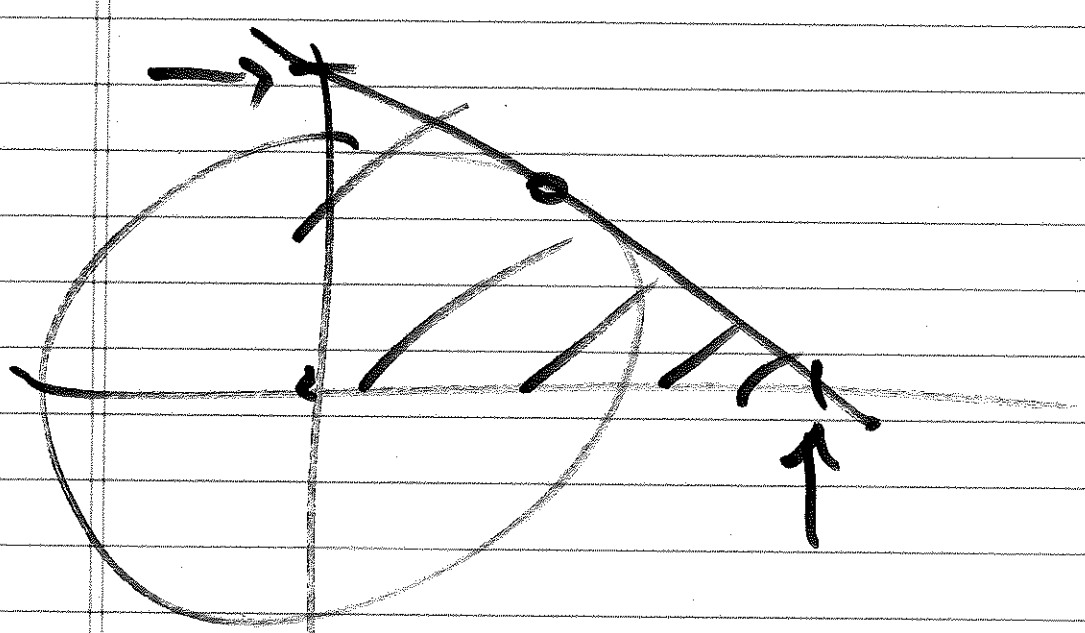


$$y - \sin t = m(x - \cos t)$$

$$m_{rad} = \frac{y-0}{x-0} = \frac{y}{x}$$

$$m_{tan} = -\frac{x}{y} = -\frac{\cos t}{\sin t}$$

$$y - \sin t = \frac{-\cos t}{\sin t}(x - \cos t)$$



x, y intercepts

$\frac{1}{2}bh$