Eg: $f(x) = \frac{1}{1+x}$:4

analysis

domain $x \neq -1$

asymptotes $y = 0$

$\lim_{x \to \infty} \frac{1}{1+x} = 0$

$\frac{1}{\infty}$

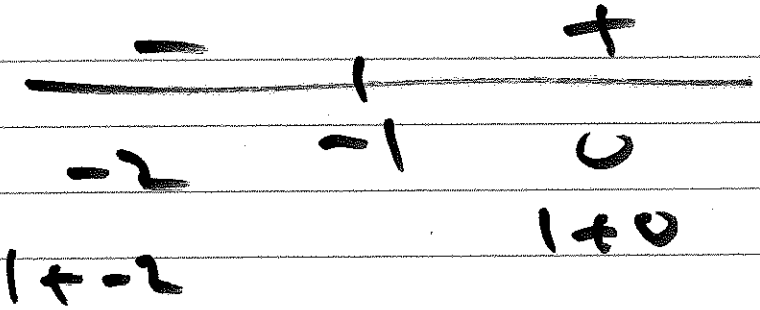
$\lim_{x \to -\infty} f(x) = 0$

$\lim_{x \to -1^+} f(x) = \pm\infty$

?

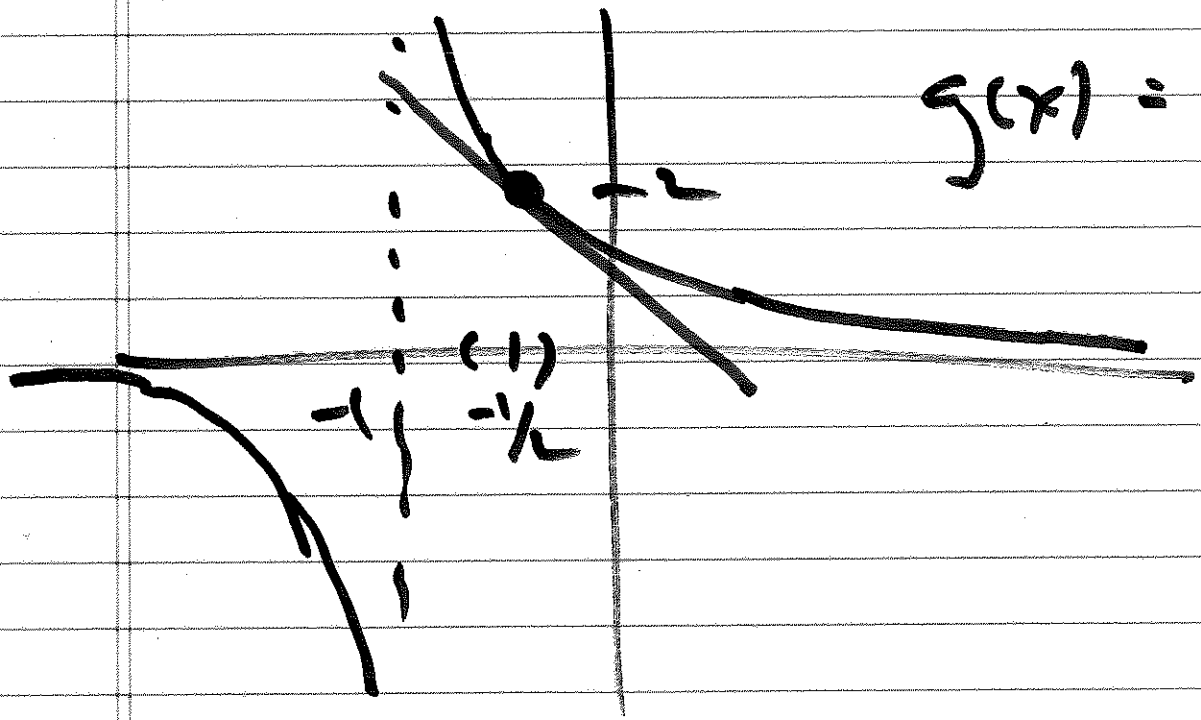
$x > 0$ is $f(x) > 0$!

Sign analysis $1+x$



$$\lim_{x \to -1+} \frac{1}{1+x} = +\infty$$

$$\lim_{x \to -1-} \frac{1}{1+x} = -\infty$$



Calculus

equation tan line

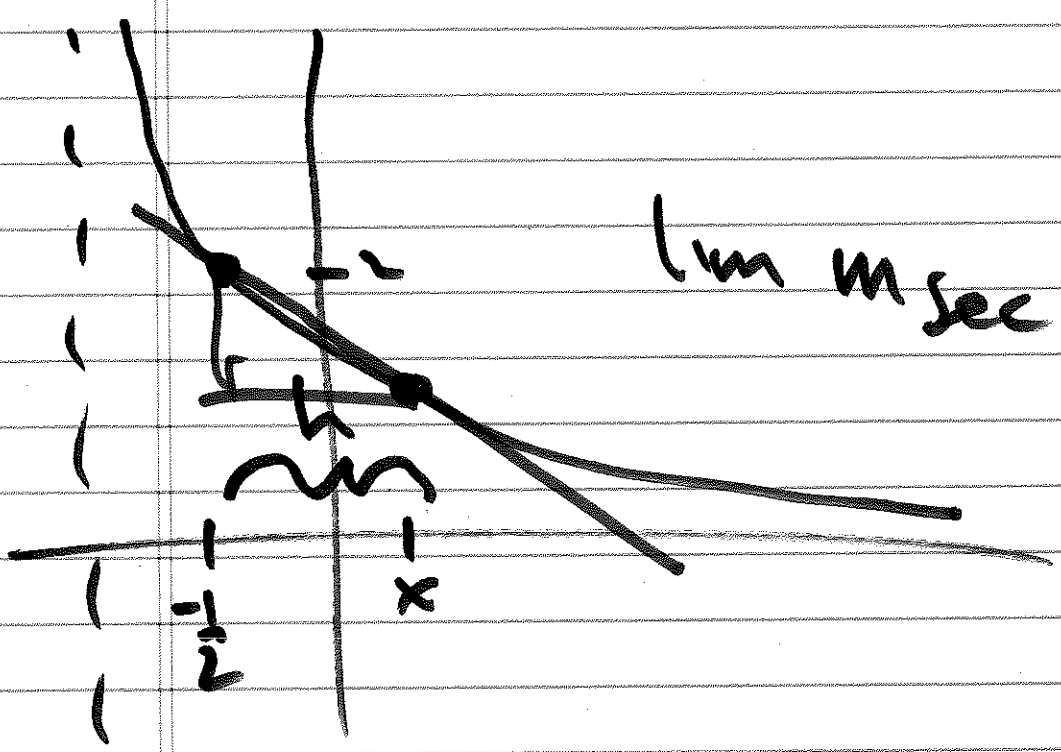
$x = -1/2$

$f(-1/2) = 2$

$y - 2 = m(x + 1/2)$

$m = m_{tan}$

$\frac{\Delta y}{\Delta x}$ $\Delta x = 0$?



① $\lim_{x \to -\frac{1}{2}} \frac{\Delta y}{\Delta x}$

2 methods

② $\lim_{h \to 0} \frac{\Delta y}{\Delta x}$ ✓

$h = \Delta x$

$\Delta y =$

$x = -\frac{1}{2} + h$

$\Delta y = f(-\frac{1}{2}+h) - f(-\frac{1}{2})$

$h - \frac{1}{2}$ $\qquad$ 2

$f(h-\frac{1}{2}) = \frac{1}{1+(h-\frac{1}{2})}$

$= \frac{1}{h+\frac{1}{2}} \frac{2}{2}$

$= \frac{2}{2h+1}$

$\Delta y = \frac{2}{2h+1} - 2$

m tan =

$$\lim_{h \to 0} \frac{\frac{2}{2h+1} - 2}{h} \cdot \frac{2h+1}{2h+1}$$

$$= \lim_{h \to 0} \frac{2 - 2(2h+1)}{h(2h+1)}$$

$$= \lim_{h \to 0} \frac{-4h}{h(2h+1)}$$

$$= \lim_{h \to 0} \frac{-4}{2h+1} = \frac{-4}{1}$$

$$y - 2 = -4(x + 1/2)$$

the derivative of $f(x)$ at $x = -1/2$ is -4

Eg: $y = -16t^2 + 31t + 2$

tomato flying

y ft

t sec

how fast at $t=1$?

$\lim_{h \to 0} \frac{\Delta y}{\Delta t}$ ft/sec

Cost Fcn

$y = C(x)$

y $

x = 1000's of tapered roller bearings

$\frac{\Delta y}{\Delta x}$ $/100 TRBs

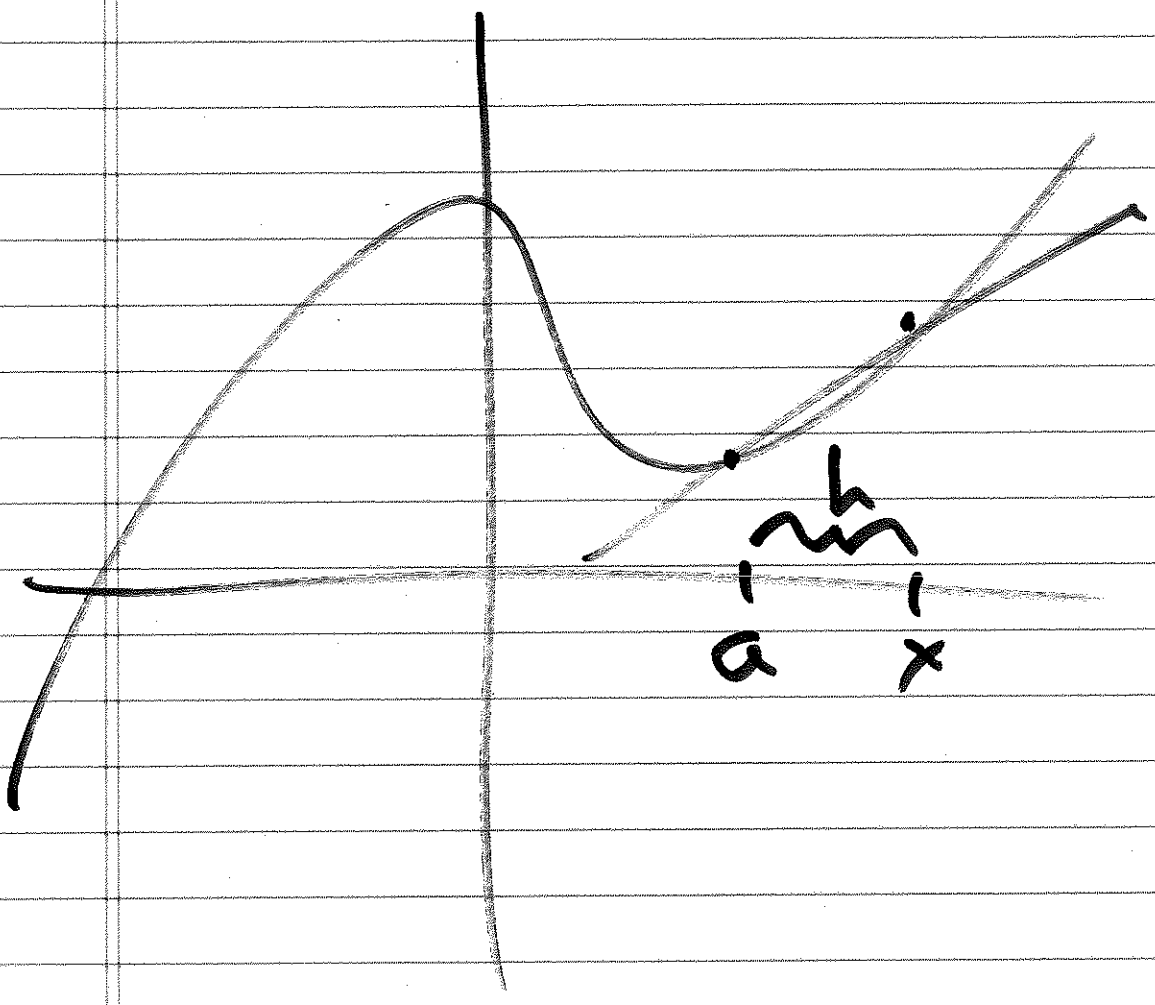
Difference Quotient

$y = f(x)$ $\quad x = a$

$$\frac{\Delta y}{\Delta x} \textcircled{1} \frac{f(x) - f(a)}{x - a} \quad \lim_{x \to a}$$

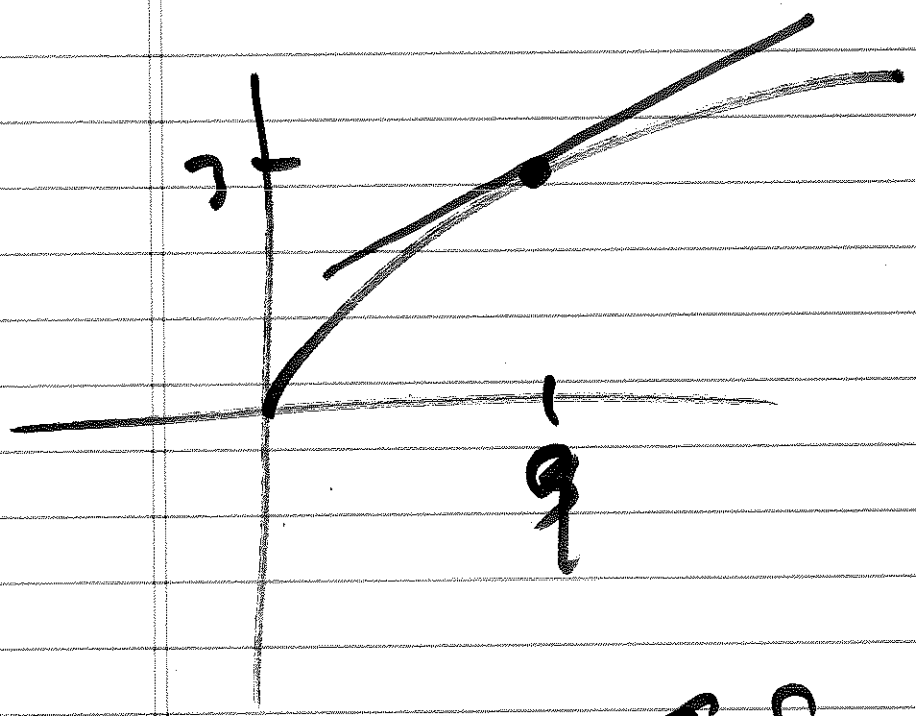
$$\textcircled{2} \frac{f(a+h) - f(a)}{h} \quad \lim_{h \to 0}$$

$h = x - a$


h
a
x

$f(x) = \sqrt{x}$ $\quad a = 9$

$f(9) = 3$



$$m_{tan} = \lim_{x \to 9} \frac{f(x) - f(9)}{x - 9}$$

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$$

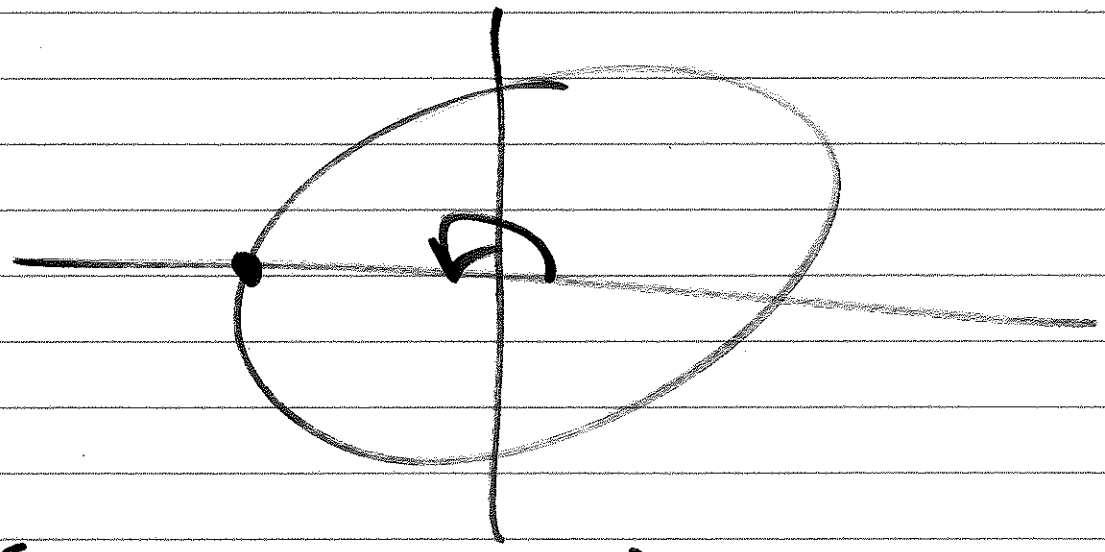
$$= \lim_{x \to 9} \frac{x - 9}{(x - 9)(\sqrt{x} + 3)}$$

$$= \lim_{x \to 9} \frac{1}{\sqrt{x} + 3} = \frac{1}{6}$$

Ex: $\lim_{x \to \pi} \frac{1 + \cos x}{x - \pi}$

$a = \pi$

$f(x) - f(\pi)$



$(\cos\pi, \sin\pi)$

$-1 \qquad 0$

$$\frac{\cos x - (-1)}{x - \pi}$$

$$\frac{\cos x - \cos\pi}{x - \pi}$$

$f(x) = \cos x \qquad (\pi, -1)$

$$g(x) = \frac{1}{1+x} \qquad x = -\frac{1}{2}$$

$$g\left(\left[h-\tfrac{1}{2}\right]\right) = \frac{1}{1+\left[h-\tfrac{1}{2}\right]}$$

try $f(x) = \cos x$ $\quad a = \pi$ given

$$\frac{f(x) - f(a)}{x - a}$$ check

$$\frac{\cos x - (-1)}{x - \pi}$$