Finish Limits

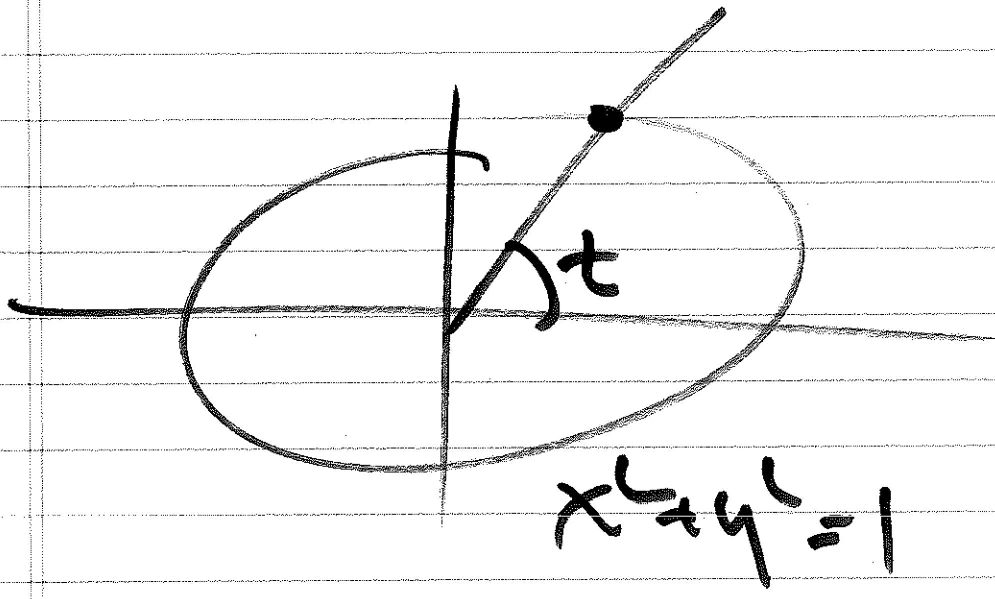
Derivatives

Continuous Fcns

Eg: $\frac{0}{0}$

$$\lim_{t \to \pi/2} \frac{\sin t - \sqrt{\sin^2 t + 2\cos^2 t}}{2\cos^2 t}.$$

$$= \lim_{t \to \pi/2} \frac{\sin^2 t - (\sin^2 t + 2\cos^2 t)}{2\cos^2 t \cdot \left[\sin t + \sqrt{\cdots}\right]}$$

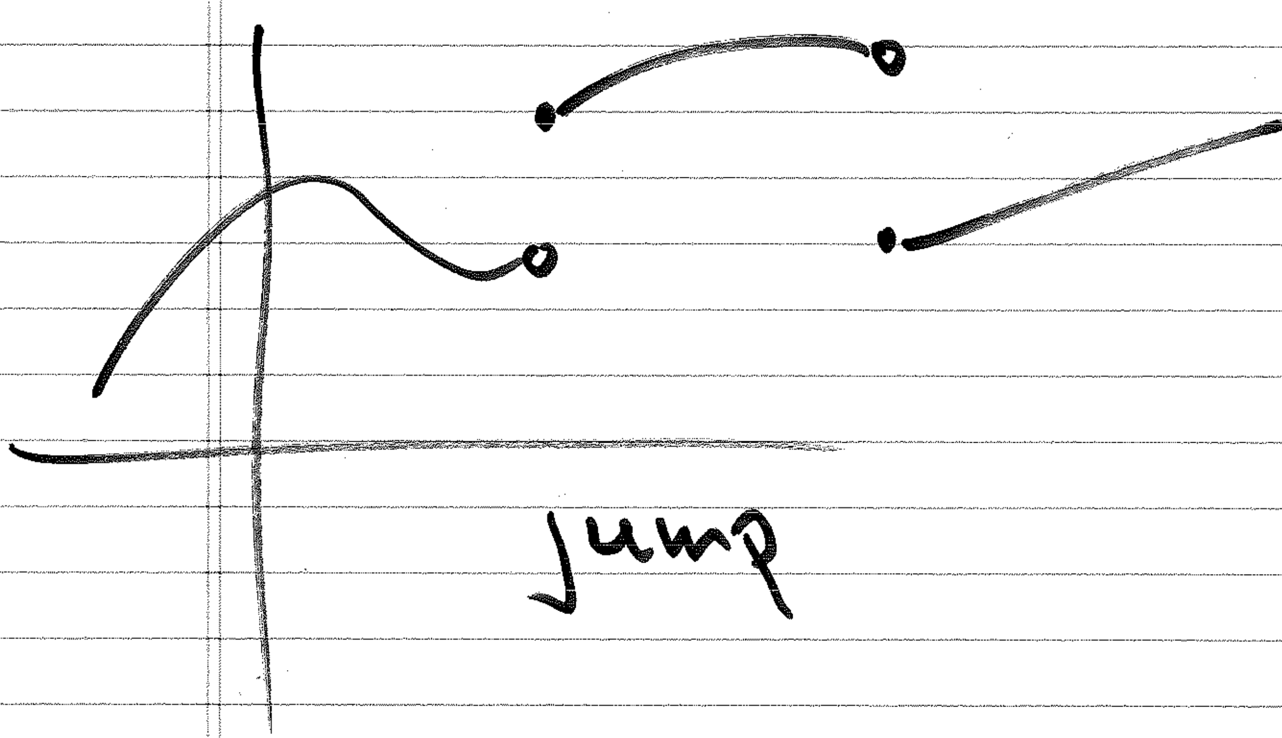


$$\frac{\sin t + \sqrt{m}}{\sin t + \sqrt{m}}$$

$$= \lim_{t \to \pi/2} \frac{-2\cos^2 t}{2\cos^2 t \cdot (\sin t + \sqrt{m})}$$

Continuous

Discontinuous

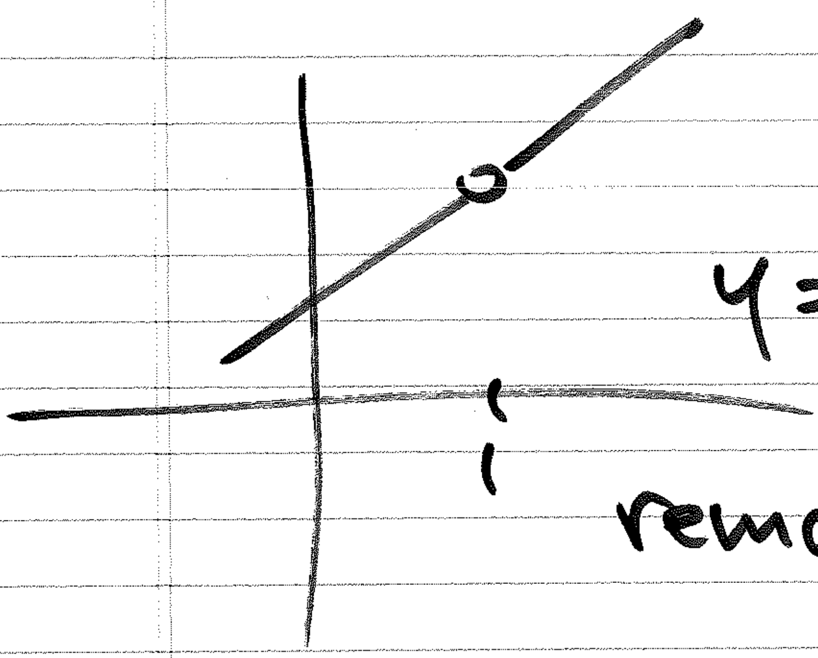


Eg: $\dfrac{x^2-1}{x-1} = f(x)$

$x \neq 1$

$$\frac{(x-1)(x+1)}{x-1}$$

$x+1 = f(x)$

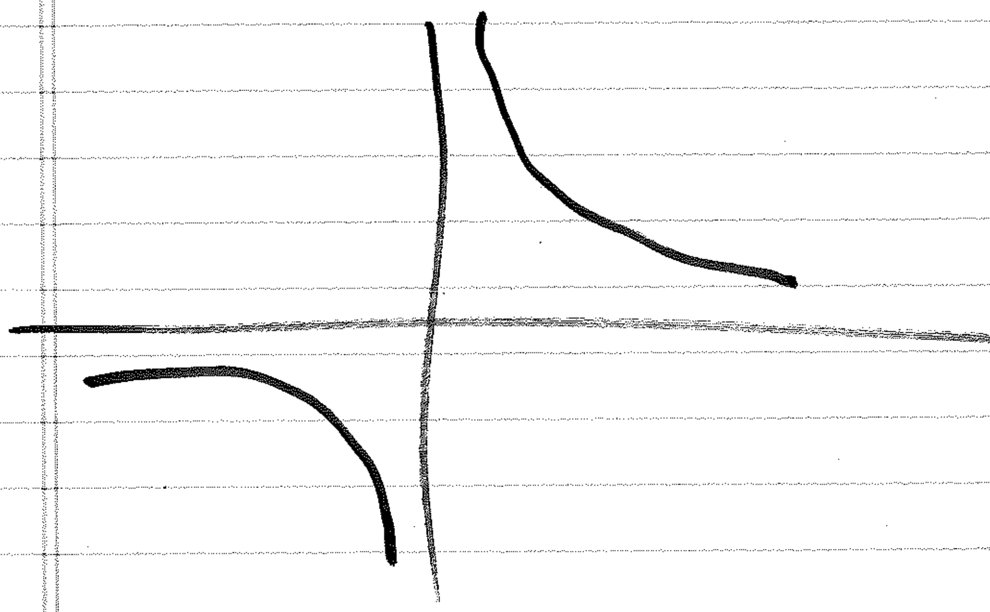


$y = f(x)$

removable

$f(x) = \frac{1}{x}$

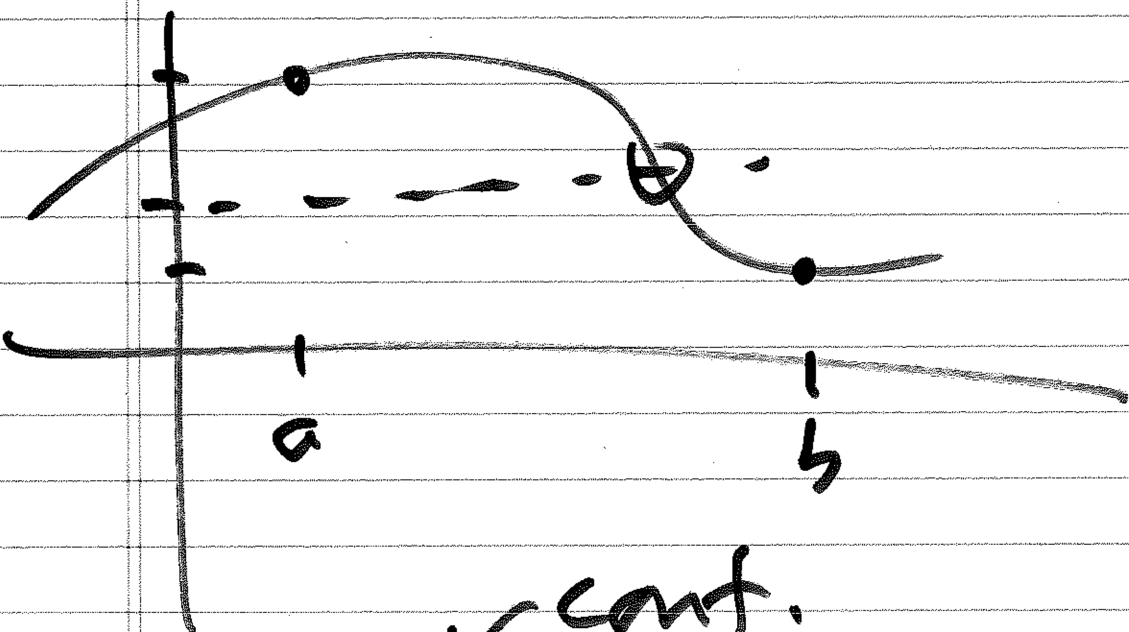
infinite



$h(x) = \sin\frac{1}{x}$

oscillation

Intermediate Value Thm



↙ cont.

if $f(a) < c < f(b)$

then there exists

$a < d < b$

with $f(d) = c$

Eg: consider $\sqrt{29}$

$f(x) = x^2 - 29$

polynomial → cont.

$f(5) = -4$

$f(6) = 7$

$-4 < 0 < 7$

So we know there's a number d so that $f(d) = 0$

Limits at infinity
horizontal asymptotes

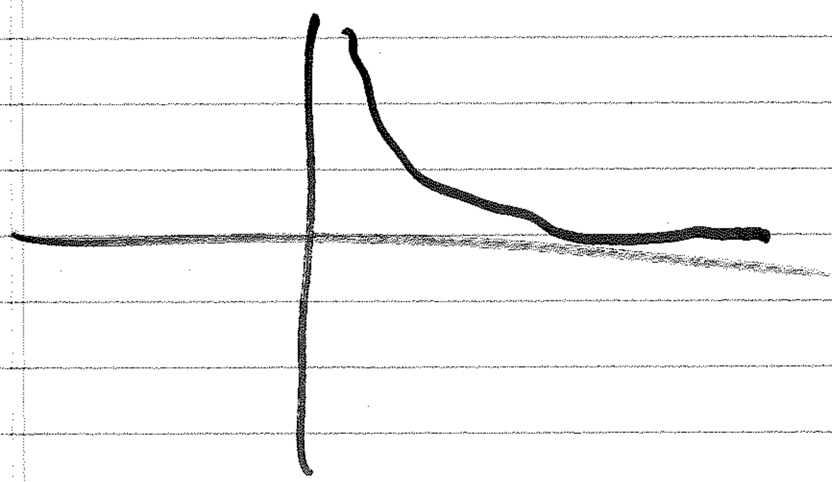
Eg: $\lim_{x \to \infty} \frac{x^2+1}{2x-7} \quad \frac{1/x^2}{1/x^2} \quad \frac{1/x}{1/x}$

$= \lim_{x \to \infty} 1 + 1/x^2$

$= \lim_{x \to \infty} \frac{x + 1/x}{2 - 7/x} = +\infty$

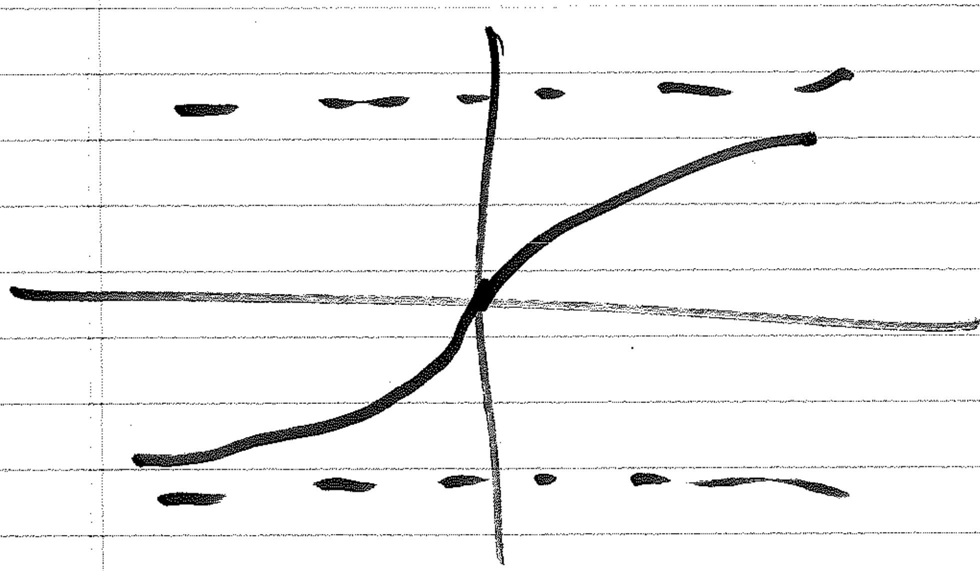
$1/x \to 0$

$7/x \to 0$



$$\frac{1}{A+B} = \frac{1}{A} + \frac{1}{B}$$

Eg: $\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$



$$\lim_{x \to \infty} \sqrt{\frac{3x^2+2x}{5-7x^2}}$$

$$= \sqrt{\lim_{x \to \infty} \frac{3x^2+2x}{5-7x^2}}$$

$$\lim_{x \to \infty} \frac{3x^2+2x}{5-7x^2} \cdot \frac{1/x^2}{1/x^2}$$

$$= \lim_{x \to \infty} \frac{3 + 2/x}{5/x^2 - 7} = -\frac{3}{7}$$

$2/x \to 0$, $5/x^2 \to 0$

$\sqrt{-3/7}$? divergent

DNE

$$\sqrt{x^2+4x} \cdot \frac{x}{\sqrt{\frac{1}{x^2}}}$$

$$\sqrt{(x^2+4x) \cdot \frac{1}{x^2}}$$

$$\sqrt{A} \cdot \sqrt{B} = \sqrt{AB}$$

Eg: $\lim\limits_{x \to -\infty} x + \sqrt{x^2+2x}$

$$= \lim_{x \to -\infty} \left(x + \sqrt{x^2+2x}\right)\left(\frac{x - \sqrt{x^2+2x}}{x - \sqrt{}}\right)$$

$$= \lim_{x \to -\infty} \frac{x^2 - (x^2+2x)}{x - \sqrt{x^2+2x}}$$

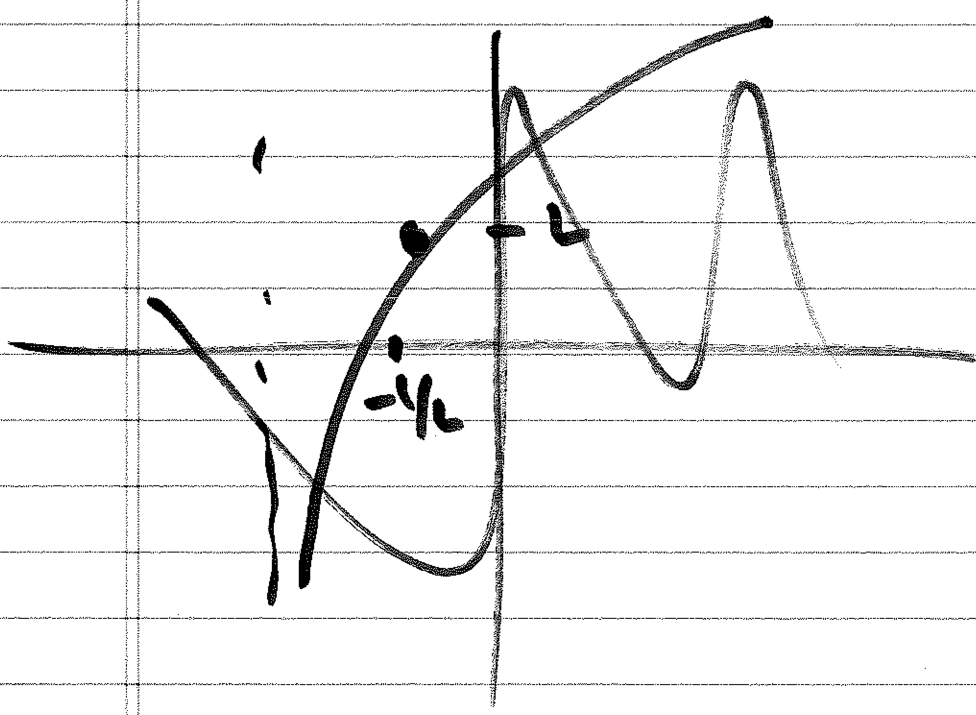
$$= \lim_{x \to -\infty} \frac{-2x}{x - \sqrt{x^2+2x}} \qquad \frac{1/x}{1/x}$$

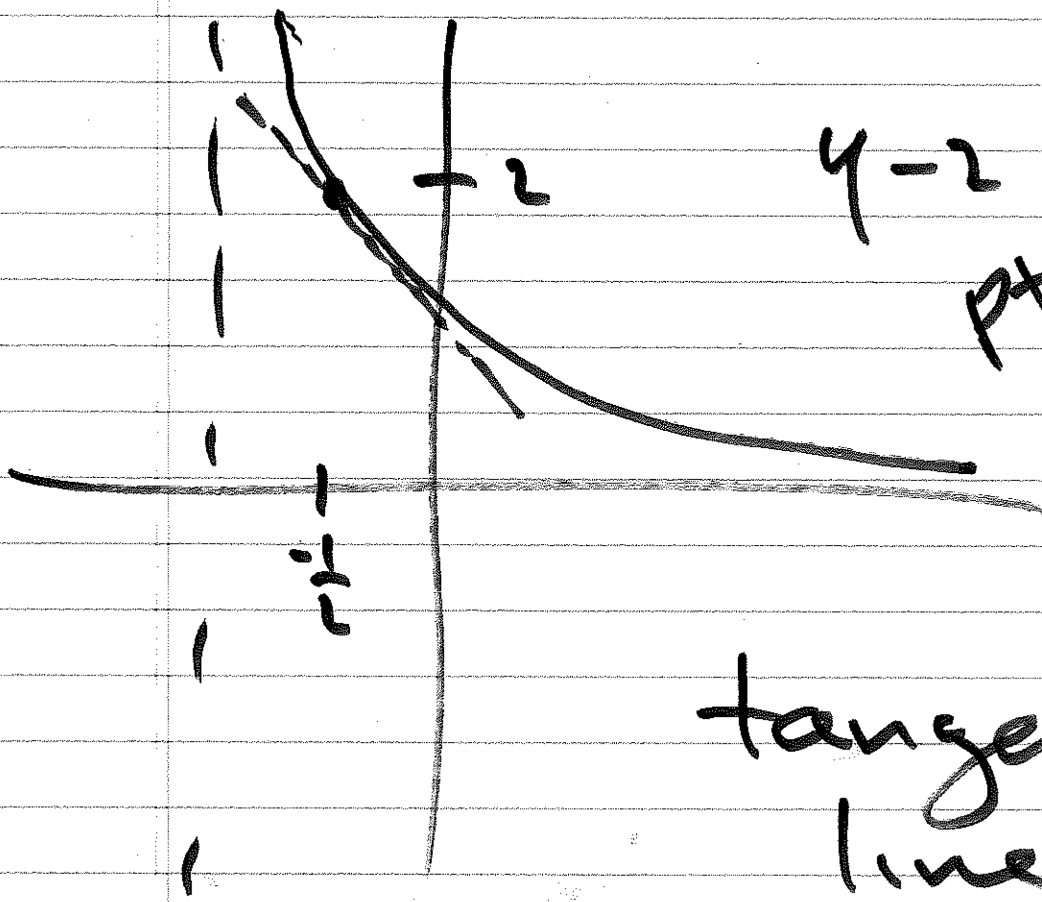
$$= \lim_{x \to -\infty} \frac{-2}{1 - \sqrt{1 + \frac{2}{x}}} \qquad \frac{-2}{0}$$

Derivatives

$$g(x) = \frac{1}{1+x}$$

$$g(-1/2) = \frac{1}{1-1/2} = 2$$

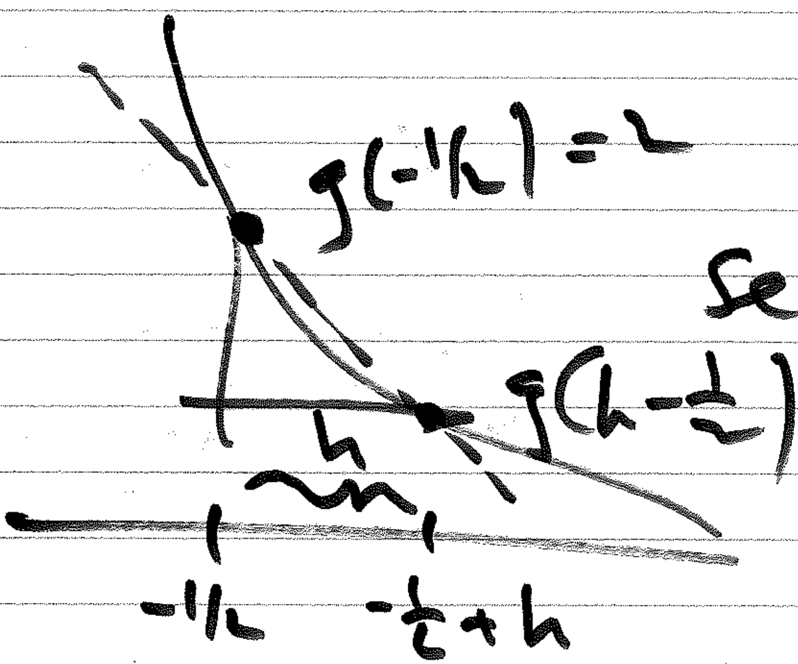




$y - 2 = m\left(x + \frac{1}{2}\right)$

pt slope

tangent line



secant line

$$g(x) = \frac{1}{1+x}$$

$$m_{sec} = \frac{\Delta y}{\Delta x} = \frac{g(h-\frac{1}{2}) - g(-\frac{1}{2})}{h}$$

$$m = m_{tan} = \lim_{h \to 0} m_{sec}$$

$$= \lim_{h \to 0} \frac{\frac{1}{1+h-\frac{1}{2}} - 2}{h}$$