Limits

Continuity

$$\lim_{x \to a} f(x) = f(a)$$

Eg: $\lim_{x \to 5} \ln(x-2) = \ln(5-2)$

$= \ln 3$

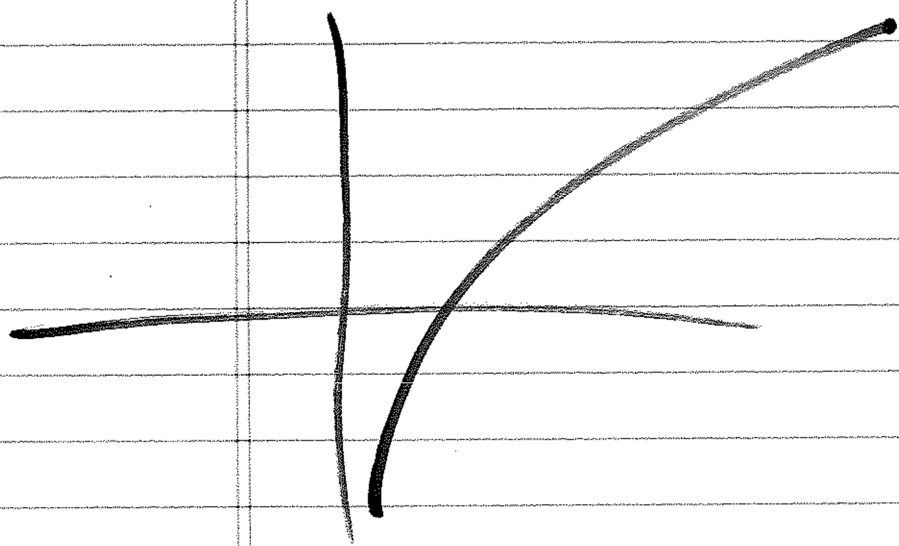
Eg: $\lim_{x \to 1^-} \ln(1-x)$

$\ln(1-x)$ $\qquad$ $\ln(1-x)$

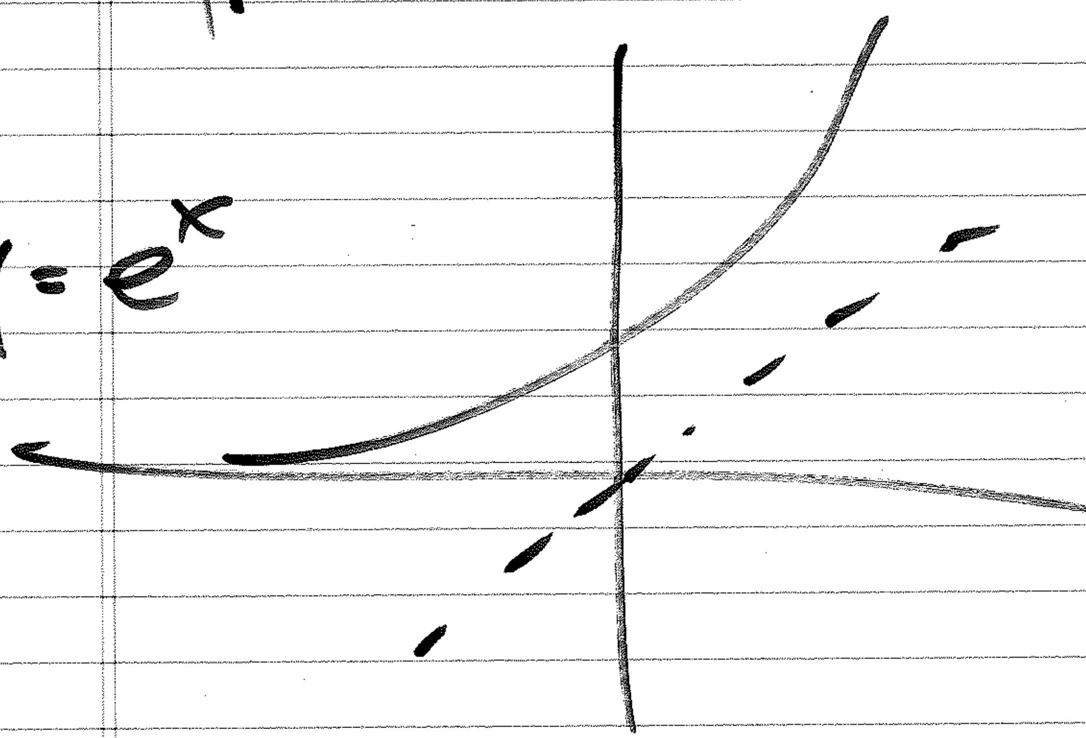
$+ \quad | \quad -$

$1-x$

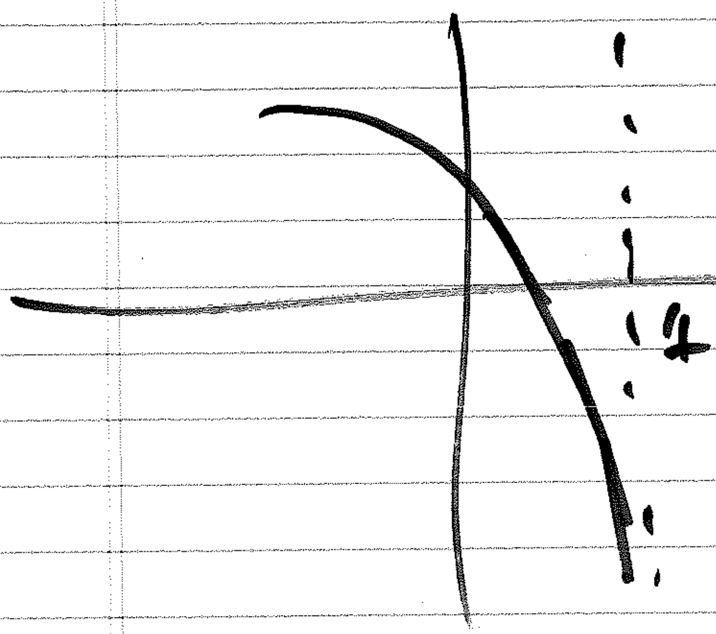
$y = \ln x$



$y = e^x$



$\ln(1-x)$



$$\lim_{x \to 1^-} \ln(1-x) = -\infty$$

∞

Limit Laws

$$\lim_{x \to a} f(x) + g(x) =$$

$$\lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

if both converge

etc ...

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

need $\lim g(x) \neq 0$

Continuity ②

Eg: $\lim_{x\to 1} \sqrt{\frac{x^2+2x-3}{x-1}} = \sqrt{4} = 2$

because $\sqrt{x}$ is cont.

$= \sqrt{\lim_{x\to 1} \frac{x^2+2x-3}{x-1}}$

① $\lim_{x\to 1} \frac{x^2+2x-3}{x-1}$ $\qquad \frac{0}{0}$

$= \lim_{x\to 1} \frac{(x+3)\cancel{(x-1)}}{\cancel{x-1}}$

$= \lim_{x\to 1} x+3 = 4$

$$\lim_{x\to 5}\left(\ln x + \cos\sqrt{\frac{2}{x+1}}\right)$$

cont at $x=5$ $= \ln 5 + \cos\sqrt{\frac{2}{6}}$

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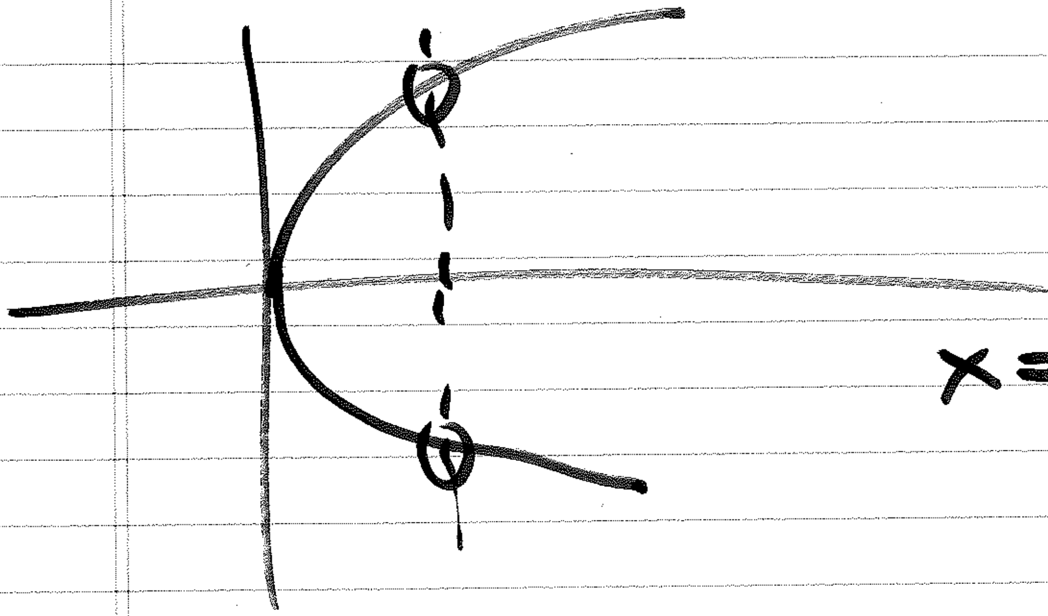
cos cont

$\sqrt{\ }$ cont

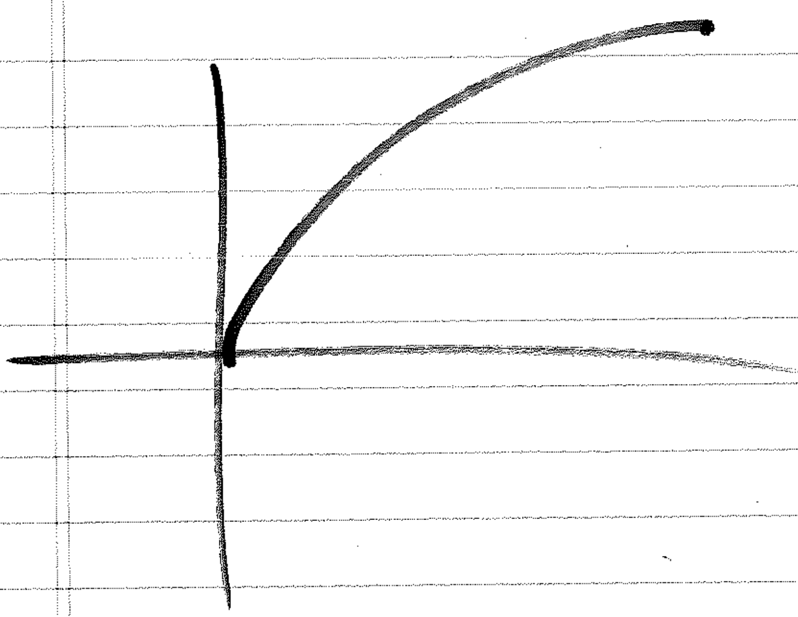
$$\lim_{x\to 5} \cos\sqrt{\frac{2}{x+1}}$$

$$\cos\sqrt{\lim \frac{2}{x+1}}$$



$x = y^2$

$f(x) = \sqrt{x}$



Eg: $f(x) = \begin{cases} 2x+3 & x<2 \\ 3x^2+ax+1 & x \geq 2 \end{cases}$

a, b, c constants

x, y, z variables

Q: for what values of a is f(x) cont?

~~lim~~

① cont if $x \neq 2$

② $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 2x+3 = 7$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 3x^2 + ax + 1 = 13 + 2a$

$13 + 2a = 7$

$a = -3$

Conjugation

Ex: $\lim_{x \to 8} \dfrac{\sqrt{12-x} - 2}{\sqrt{24-x} - 4} = \dfrac{0}{0}$

Alg

$$\frac{\left(\sqrt{12-x} - 2\right)}{\sqrt{24-x} - 4} \cdot \frac{\left(\sqrt{12-x} + 2\right)}{\sqrt{12-x} + 2}$$

$$\frac{12 - x - 4}{\left(\sqrt{24-x} - 4\right)\left(\sqrt{12-x} + 2\right)}$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$\frac{8-x}{\left(\sqrt{24-x} - 4\right)\left(\sqrt{12-x} + 2\right)} \qquad \frac{0}{0}$$

$$\frac{8-x}{\left(\sqrt{24-x} - 4\right)\left(\sqrt{12-x} + 2\right)} \cdot \frac{\sqrt{24-x} + 4}{\sqrt{24-x} + 4}$$

$$\frac{\cancel{(8-x)}\left(\sqrt{24-x} + 4\right)}{\cancel{(24-x-16)}\left(\sqrt{12-x} + 2\right)}$$

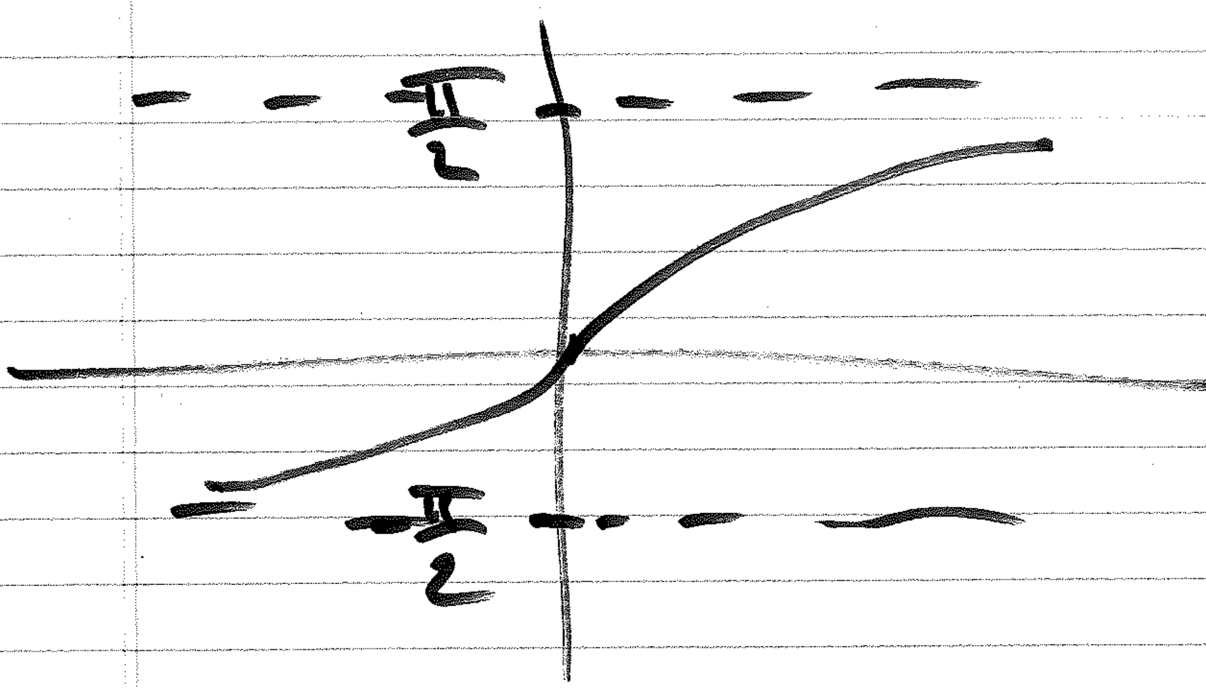
$$* = \lim_{x \to 8} \frac{\sqrt{24-x} + 4}{\sqrt{12-x} + 2} = \frac{4+4}{2+2} = 2$$

Squeeze Thm

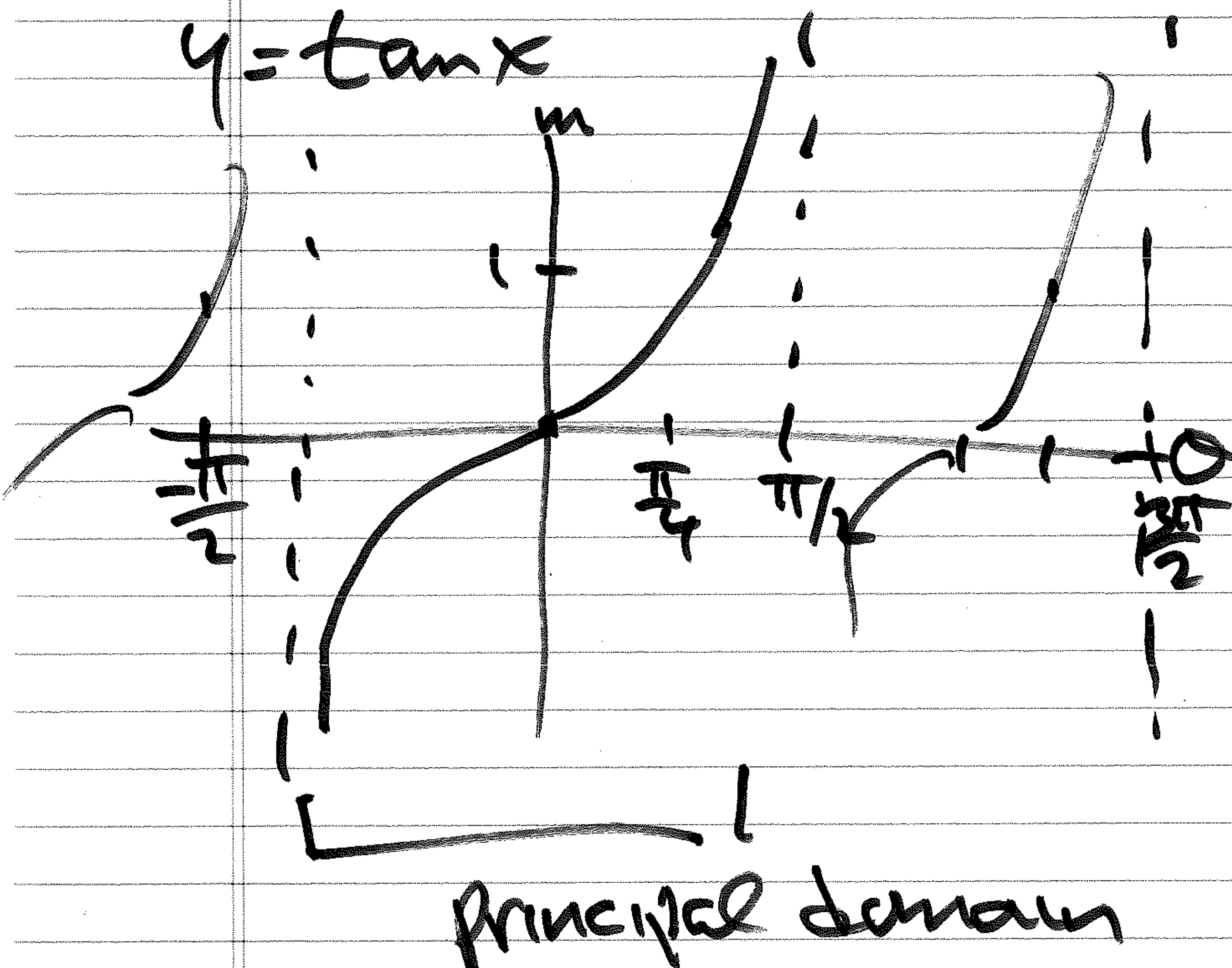
Sandwitch Thm

Eg: $\lim_{x \to 0} x^3 \cdot \tan^{-1}(\ln x) = 0$

$y = \tan^{-1} x$



$y = \tan^{-1} x$



$$-\frac{\pi}{2} < \tan^{-1} \ln x \leq \frac{\pi}{2}$$

$$x^3 \cdot -\frac{\pi}{2} \leq x^3 \tan^{-1} \ln x \leq x^3 \cdot \frac{\pi}{2}$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$0 \qquad\qquad\qquad\qquad 0$$

$$\lim_{x \to 0} \frac{\pi}{2} x^3 = 0$$