1. (13 total points) Let $f(x) = 5x + |3x-2|$ and $g(x) = x^2 + 4x$.

   (a) (7 points) Find all solutions to the equation $f(x) = 14$

   Write $f(x) = \begin{cases} 5x + (3x-2) & \text{if } x \geq \frac{2}{3}; \\ 5x - (3x-2) & \text{if } x < \frac{2}{3}. \end{cases}$

   Solve $14 = 5x + (3x-2)$ *to get* $x = 2$. *This is a solution, because* $2 \geq \frac{2}{3}$ *is true.*

   Solve $14 = 5x - (3x-2)$ *to get* $x = 6$. *This is not a solution, because* $6 < \frac{2}{3}$ *is false.*

   The only solution is $x = 2$.

   (b) (6 points) Calculate a formula for $g(3x-2)$. Simplify as much as possible.

$$\begin{aligned} g(3x-2) &= (3x-2)^2 + 4(3x-2) \\ &= 9x^2 - 12x + 4 + 12x - 8 \\ &= 9x^2 - 4 \end{aligned}$$

2. (12 Tafu is an Econ professor. In 2011 he earned \$96,000 and in 2014 he earned \$101,000. Nell fries chicken at Ezell's. In 2012 she earned \$27,000 and in 2016 she earned \$32,000. Take $t = 0$ in 0CE (also known as 0AD).

   (a) (4 points) Give a linear function relating Tafu's salary T to the year t.

   Let T be Tafu's salary in units of \$1000. The two points are $(2011, 96)$ *and* $(2014, 101)$.
   $\Delta T = 5$ *and* $\Delta t = 3$ *so the slope is* $m = \frac{5}{3}$.
   The line is $T - 96 = \frac{5}{3}(t - 2011)$.
   The function is $T(t) = 96 + \frac{5}{3}(t - 2011)$.

   (b) (4 points) Give a linear function relating Nell's salary N to the year t.

   Let N be Nell's salary in units of \$1000. The two points are $(2012, 27)$ *and* $(2016, 32)$.
   $\Delta N = 5$ *and* $\Delta t = 4$ *so the slope is* $m = \frac{5}{4}$.
   The line is $N - 27 = \frac{5}{4}(t - 2012)$.
   The function is $N(t) = 27 + \frac{5}{4}(t - 2012)$.

   (c) (4 points) In what year will Tafu earn three times as much as Nell?

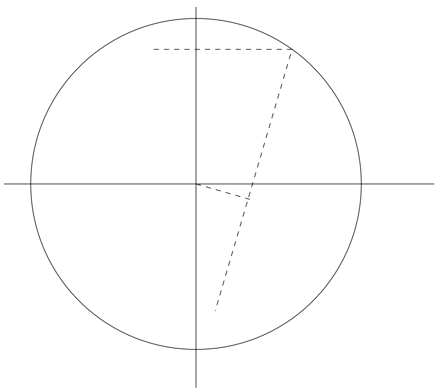
$$\begin{aligned}
T &= 3N \\
96 + \frac{5}{3}(t - 2011) &= 3 \cdot \left[27 + \frac{5}{4}(t - 2012)\right] \\
96 + \frac{5}{3}(t - 2011) &= 81 + \frac{15}{4}(t - 2012) \\
\frac{5}{3}(t - 2011) &= \frac{15}{4}(t - 2012) - 15 \\
20(t - 2011) &= 45(t - 2012) - 180 \\
t &= 2020
\end{aligned}$$

   In the year 2020.

3. (13 points) Clovis stands 4 feet West and 12 feet North of an apple tree. Ida stands 2 feet East and 9 feet South of the tree.

   Clovis walks due East until he is 13 feet away from the tree. Then, he turns and walks in a straight line towards Ida.

   How close does Clovis get to the tree?



Take the origin at the apple tree, as in the figure.

First compute where Clovis turns. This is the point where the line $y = 12$ intersects the circle $x^2 + y^2 = 169$.

$$\begin{aligned} x^2 + (12)^2 &= 169 \\ x^2 &= 25 \\ x &= \pm 5 \end{aligned}$$

The point is East of his starting point $(-4, 12)$ so it must be $(5, 12)$.

Next, compute his path to Ida. This is the line from $(5, 12)$ to $(2, -9)$. The slope is $m = 7$ and the line is $y + 9 = 7(x - 2)$.

Now compute the point on the path closest to the tree. The line through the origin perpendicular to the path is $y = -\frac{1}{7}x$. We compute the intersection:

$$\begin{aligned} -\frac{1}{7}x &= -9 + 7(x - 2) \\ -\frac{1}{7}x &= 7x - 23 \\ -x &= 49x - 161 \\ x &= 3.22 \end{aligned}$$

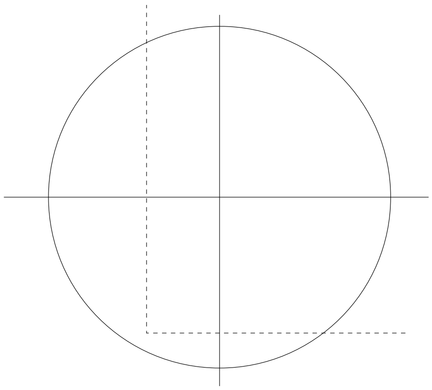
Thus $x = 3.22$. We can use $y = -\frac{1}{7}x$ to compute $y = -0.46$.

Finally, we compute the distance to the origin: $\sqrt{(3.22)^2 + (-0.46)^2} \approx 3.25$ *feet.*

4. (12 points) Tafu is sailing near a radar buoy which can detect anything within 10 km of the buoy. He starts sailing from a point 12 km East and 8 km South of the buoy. He sails West for two hours, then turns and sails North for 25 km.

   He sails at a constant speed of 8.5 km/hr.

   How much time was he within 10 km of the buoy?



Put the origin of your coordinate system at the position of the buoy. It can detect anything within the circle $x^2+y^2=100$

Tafu's path at first follows the line $y=-8$. *We wish to intersect it with the circle.*

$$\begin{aligned} x^2+(-8)^2 &= 100 \\ x^2 &= 36 \\ x &= \pm 6 \end{aligned}$$

Thus Tafu enters the circle at the point $(6,-8)$.

He travels 17 km in 2 hours. This puts him at the point $(-5,-8)$. *He has gone 11 km inside the circle.*

Now his path follows the line $x=-5$. *We intersect it with the circle to find where he exits.*

$$\begin{aligned} (-5)^2+y^2 &= 100 \\ y^2 &= 75 \\ y &= \pm 5\sqrt{3} \end{aligned}$$

Thus Tafu exits the circle at the point $(-5,5\sqrt{3})$. *He has gone an additional* $8+5\sqrt{3}$ *km in the circle.*

He travels a total of $19+5\sqrt{3}$ *km in the circle.*

The amount of time spent is $\dfrac{19+5\sqrt{3}}{8.5} \approx 3.25$ *hours.*