1. (6 points) Let $f(x) = x^2 - 2x + 3 \cdot 4^x$. Suppose we take the graph of $y = f(x)$ and do three things:

   - First, expand it horizontally by a factor of 3.
   - Then, shift it 5 units to the left.
   - Then, shift it 7 units up.

   Write a function $g(x)$ for this new transformed graph.

$$\begin{aligned} g(x) &= f\left(\frac{1}{3}(x+5)\right) + 7 \\ &= \left(\frac{1}{3}(x+5)\right)^2 - 2\left(\frac{1}{3}(x+5)\right) + 3 \cdot 4^{\frac{1}{3}(x+5)} + 7 \end{aligned}$$

2. (6 points) Let $F(x) = 3x - 4$ and $H(x) = 2x^2 + x$. Find the fixed points of $F \circ H(x)$.

$$\begin{aligned} F \circ H(x) &= 3\left(2x^2 + x\right) - 4 \\ &= 6x^2 + 3x - 4 \end{aligned}$$

$$\begin{aligned} F \circ H(x) &= x \\ 6x^2 + 3x - 4 &= x \\ 6x^2 + 2x - 4 &= 0 \\ 2(3x-2)(x+1) &= 0 \end{aligned}$$

$$x = -1, \frac{2}{3}$$

3. (13) The average monthly rent for a one-bedroom apartment in Honolulu is growing exponentially.

   (a) (4 points) In the year 2010, the rent in Honolulu was \$1200 a month and it has an annual growth rate of 3.5%.
   Write a function $f(t)$ for the rent in Honolulu t years after 2010.

   $f(t) = 1200 \cdot (1.035)^t$

   (b) (5 points) The average monthly rent in Kona is also growing exponentially.
   In the year 2012, the rent in Kona was \$500 less than the rent in Honolulu.
   In the year 2018, the rent in Kona is \$1000.
   Write a function $g(t)$ for the rent in Kona t years after 2010.

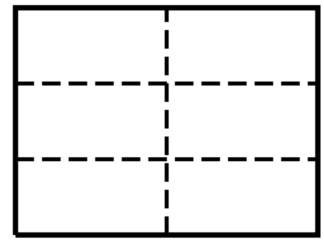
$$\begin{aligned}
g(t) &= A \cdot b^t \\
g(2) &= f(2) - 500 \\
&= 785.47 \\
b^6 &= \frac{g(8)}{g(2)} \\
&= \frac{1000}{785.47} \\
b &= \left(\frac{1000}{785.47}\right)^{1/6} \\
1000 &= A \cdot \left(\frac{1000}{785.47}\right)^{8/6} \\
A &\approx 724.72 \\
\\
g(t) &= 724.72 \cdot \left(\frac{1000}{785.47}\right)^{t/6}
\end{aligned}$$

   (c) (4 points) When will the rents in Honolulu and Kona be equal?
   (Round your answer to the nearest year.)

$$\begin{aligned}
1200 \cdot (1.035)^t &= 724.72 \cdot \left(\frac{1000}{785.47}\right)^{t/6} \\
\ln(1200) + t \cdot \ln(1.035) &= \ln(724.72) + t \cdot \frac{1}{6} \cdot \ln\left(\frac{1000}{785.47}\right) \\
t &= \frac{\ln(1200) - \ln(724.72)}{\frac{1}{6} \cdot \ln\left(\frac{1000}{785.47}\right) - \ln(1.035)} \\
&\approx 86.3
\end{aligned}$$

   In the year 2096.

4. (13 points) Isobel would like to build a rectangular enclosure which will be divided into six sections in a 3×2 configuration, as shown below.
   The material for the outside fence costs \$10 per foot, and the material for the inner partitions costs \$6 per foot.
   If Isobel has \$3900 total to spend, what is the **maximum possible area** of the enclosure?



Let x be the width of the enclosure and y be the length.

We wish to maximize $A = xy$.

The length of the outside fence is $2x + 2y$.

The total length of the inner partitions is $2x + y$.

The total cost of the material is $10 \cdot (2x + 2y) + 6 \cdot (2x + y)$.

Thus the constraint is $32x + 26y = 3900$. *We can write this* $y = \dfrac{1950 - 16x}{13}$.

$$\begin{aligned} \text{Max } A &= xy \\ &= x \cdot \left(\frac{1950 - 16x}{13} \right) \\ &= -\frac{16}{13} x^2 + 150x \end{aligned}$$

The x-coordinate of the vertex is given by

$$\begin{aligned} x_v &= -\frac{b}{2a} \\ &= -\frac{150}{2 \cdot (-16/13)} \\ &= 60.9375 \end{aligned}$$

The maximum area is

$$A = -\frac{16}{13} \cdot (60.9375)^2 + 150 \cdot (60.9375) = 4570.3125 \text{ ft}^2$$

5. (12) The older Tafu gets, the taller he gets. At the age of 10, he was 5 feet tall. At the age of 20, he was 6 feet tall. He will always get taller, and his height will approach, but never exceed 6 feet 6 inches.

   Suppose his height is a linear-to-linear rational function of time.

   (a) (8 points) Find a function that gives his height as a function of time.

   Let y be Tafu's height in feet, and t be his age in years.
   The general model is $y = \dfrac{at+b}{t+c}$.
   We are given the horizontal asymptote $y = 6.5$. *We conclude that* $a = 6.5$.

$$\begin{aligned} 5 &= \frac{6.5 \cdot 10 + b}{10 + c} \\ 50 + 5c &= 65 + b \\ 5c - b &= 15 \end{aligned}$$

$$\begin{aligned} 6 &= \frac{6.5 \cdot 20 + b}{20 + c} \\ 120 + 6c &= 130 + b \\ 6c - b &= 10 \end{aligned}$$

   Subtracting the first equation from the second gives $c = -5$. *Then* $b = 6c - 10 = -40$.
   The function is $y = \dfrac{6.5t - 40}{t - 5}$

   (b) (4 points) According to your model, at what age will Tafu be 6 feet 3 inches tall?

$$\begin{aligned} 6.25 &= \frac{6.5t - 40}{t - 5} \\ 6.25t - 31.25 &= 6.5t - 40 \\ 8.75 &= 0.25t \\ t &= 35 \textit{ years old} \end{aligned}$$