1. (13 total points) A farming village in Ukraine produces cabbages and pigs. In 2005, they had 200 cabbages and 10 pigs. In 2012, they had 305 cabbages and 19 pigs.

   In this problem, take $t = 0$ in 2005.

   (a) (4 points) Give a linear function relating the number of cabbages C to the year t.

   $$\Delta t = 7, \quad \Delta C = 105, \quad m = \frac{105}{7} = 15$$
   $$C - 200 = 15 \cdot (t - 0)$$
   $$C = 15t + 200$$

   (b) (4 points) Give a linear function relating the number of pigs P to the year t.

   $$\Delta t = 7, \quad \Delta P = 9, \quad m = \frac{9}{7}$$
   $$P - 10 = \frac{9}{7} \cdot (t - 0)$$
   $$P = \frac{9}{7}t + 10$$

   (c) (5 points) According to your model, in what year were there 23 cabbages for every pig?

   We need to solve the equation $C = 23P$.

   $$\begin{aligned} 15t + 200 &= 23 \cdot \left(\frac{9}{7}t + 10\right) \\ 105t + 1400 &= 207t + 1610 \\ -210 &= 102t \\ t &\approx -2.05 \end{aligned}$$

   In the year 2002.

2. (14 total points) Tafu and Clovis begin walking in the xy-plane at constant speeds at the same time.

   Tafu walks from $(-3,-4)$ to $(12,2)$ in a straight line, reaching it in 18 seconds.

   Clovis walks from $(6,2)$ in a straight line. When Tafu crosses the x-axis, Clovis is at $(0,5)$.

   (a) (4 points) Write parametric equations for Tafu's position, t seconds after he starts walking.

   $\Delta x = 15, \quad \Delta y = 6, \quad \Delta t = 18$
   $v_x = \frac{5}{6}, \quad v_y = \frac{1}{3}$
   $$\begin{cases} x_T = \frac{5}{6}t - 3 \\ y_T = \frac{1}{3}t - 4 \end{cases}$$

   (b) (5 points) Write parametric equations for Clovis' position, t seconds after he starts walking.

   First compute Δt. Tafu crosses the x-axis when $y_T = 0$.

   $$\begin{aligned} y_T &= 0 \\ \frac{1}{3}t - 4 &= 0 \\ t &= 12 \end{aligned}$$

   $\Delta x = -6, \quad \Delta y = 3, \quad \Delta t = 12$
   $v_x = -\frac{1}{2}, \quad v_y = \frac{1}{4}$
   $$\begin{cases} x_C = -\frac{1}{2}t + 6 \\ y_C = \frac{1}{4}t + 2 \end{cases}$$

   (c) (5 points) At what time is Clovis directly North of Tafu?

   We need to find t so that $x_C = x_T$.

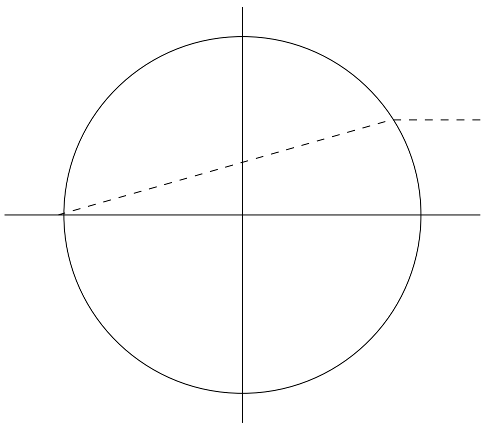
   $$\begin{aligned} x_C &= x_T \\ -\frac{1}{2}t + 6 &= \frac{5}{6}t - 3 \\ 9 &= \frac{8}{6}t \\ t &= \frac{54}{8} = 6.75\,sec \end{aligned}$$

   Note that $\quad y_C(6.75) = 3.6875 > -1.75 = y_T(6.75) \quad$ *so that Clovis is North of Tafu.*

3. (13 points) Isobel stands 30 miles east and 10 miles north of the center of a circular lake with radius 26 miles.

   She walks due west in a straight line until she hits the edge of the lake. Then, she swims in a straight line towards the westernmost point of the lake.

   When she is closest to the center of the lake, how far is she from her starting position?



We impose a coordinate system with the origin at the center of the lake.

Isobel starts at the point $(30,10)$.

Compute the point where she enters the lake.

$$\begin{aligned} x^2+y^2 &= 26^2 \\ y &= 10 \\ x^2+100 &= 676 \\ x &= \pm 24 \end{aligned}$$

She enters the lake at $(24,10)$.

Next find the equation of the path of her swim. This is the line from $(24,10)$ *to* $(-26,0)$.

$\Delta x = -50, \quad \Delta y = -10, \quad m = \frac{1}{5}$

$y = \frac{1}{5}(x+26)$

Now find the coordinates of the point on her path closest to the center of the lake.

The line through the center perpendicular to her path is $\quad y = -5x$

$$\begin{aligned} -5x &= \frac{1}{5}(x+26) \\ -25x &= x+26 \\ -26 &= 26x \\ x &= -1 \end{aligned}$$

The intersection point is $(-1,5)$. *We compute the distance from this point to the point where she started.*

$$\sqrt{(-1-30)^2+(5-10)^2} = \sqrt{986} \approx 31.4\,\mathit{miles}$$

4. (10 points) Consider the following multipart function $f(x)$:

$$f(x) = \begin{cases} 2x^2 + x & \text{if } x \leq 0 \\ 9x - 2 & \text{if } 0 < x \leq 3 \\ 7 & \text{if } x > 3 \end{cases}$$

Find all values of x such that $f(x) = 6x - 3$.

We must solve 3 equations, and check that their solutions satisfy the correct inequalities.

$$\begin{aligned} 2x^2 + x &= 6x - 3 \\ 2x^2 - 5x + 3 &= 0 \\ (2x-3)(x-1) &= 0 \\ x &= \frac{3}{2}, 1 \end{aligned} \qquad \textit{Neither value satisfies } x < 0.$$

$$\begin{aligned} 9x - 2 &= 6x - 3 \\ 3x &= -1 \\ x &= -\frac{1}{3} \end{aligned} \qquad \textit{This value is not between } 0 \textit{ and } 3.$$

$$\begin{aligned} 7 &= 6x - 3 \\ 10 &= 6x \\ x &= \frac{5}{3} \end{aligned} \qquad \textit{This value is not greater than } 3.$$

There are no solutions to this equation.