Virtual Lines in Topoland with These Designs

February 11, 2004

Abstract

Long amusement park lines significantly reduce park user's enjoyment. To mitigate this problem Topoland currently uses a QuickPass system, but it has several flaws. In this paper, we propose several QuickPass schemes in an attempt to increase people's enjoyment of the amusement park and avoid the current problems with Topoland's scheme. Using a model of our design, we implement a computer simulation to measure the enjoyment of park users for various proposed schemes. We then adjust parameters to maximize park user's enjoyment while providing users equal opportunity to park rides.

We conclude by recommending a QuickPass scheme to Topoland. Our recommended QuickPass scheme depends on the day's expected park user population. If the park occupancy is less than 11,000, we recommend that Topoland reserve 75% of the available seats on its five major rides for QuickPass users. For each ride only 25% of the seats should go to the normal line and the remaining 75% should go to the QuickPass line. If the occupancy is greater than 11,000, we recommend that each QuickPass reserve a virtual spot in the normal line. In this case, QuickPass users are loaded before normal users.

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1 Executive Summary

In recent years, amusement parks around the world have invested in *QuickPass* schemes to mitigate long wait times for popular attractions. Growth over recent years and the addition of several "thrill" rides have given Topoland much potential for future profits. However, the QuickPass scheme that is currently being used by Topoland is critically flawed. If Topoland does not improve the operation of their QuickPass scheme, Topoland risks losing customers to other parks.

In our report, we describe a model and a simulation which was used to evaluate several QuickPass schemes. The model is customizable and extensible.

Based on our simulations we recommend Topoland use the following QuickPass scheme:

1. If the number of people in the park is less than 11,000 then the park reserves 75% of seats for the QuickPass users and the remaining 25% for normal users.
2. If the number of people in the park is greater than 11,000, then each QuickPass reserves a virtual spot in line, as if a friend would hold a position for the user.

In both cases, a person will be given a return time that is the sum of the normal line's wait time and the time to serve users with QuickPasses having earlier return times.

Our proposed scheme

1. maximizes consumer enjoyment
2. gives each park customer an equal opportunity
3. and is easy to implement and maintain

We are committed to any future changes to the QuickPass system.

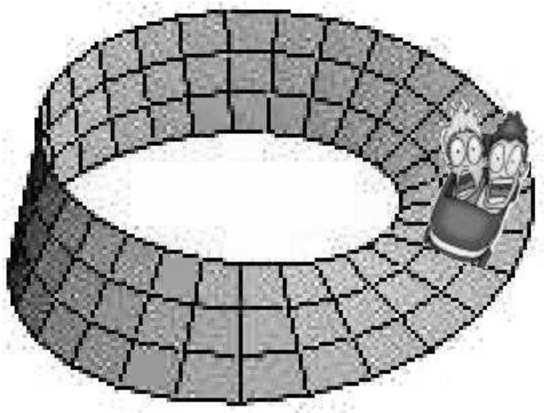

Figure 1: The Möbius Madness Roller Coaster

2 Introduction

After the introduction of the popular *Möbius Madness Roller Coaster*, which circles the track twice to return riders right side up, the waiting times at the *Topoland* amusement park became an unpleasant two hours or more. To reduce waiting time, Topoland developed and implemented a *QuickPass scheme*.

In a QuickPass scheme, a customer may decide not to wait in the long line for an attraction and instead insert their entrance ticket into a machine near the attraction. The machine gives the customer a *QuickPass* that allows him or her to return at a given later time and wait in a second line which is presumably quite short.

The QuickPass scheme currently used by Topoland has several flaws. The system may give a pass with a return time of four hours but then shortly thereafter give a pass with a return time of only one hour. Also the lines for people with passes can be as long as the normal line.

In this report we compare various QuickPass systems and recommend a QuickPass system to replace the current one used by Topoland. **Our task is the following:**

1. Propose various QuickPass schemes.
2. Determine the criteria we will use in evaluating our proposed schemes.
3. Develop and implement a model to evaluate these schemes according to our determined criteria.

Our approach is:

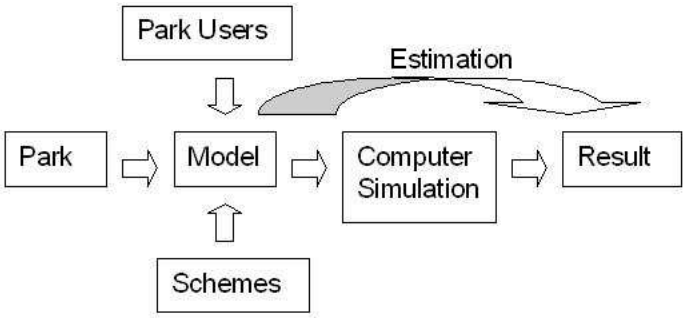

Figure 2: Structure of our Evaluations

1. We consider necessary conditions for a QuickPass scheme and propose various QuickPass schemes for evaluation. We also hypothesize the advantages and disadvantages of each of these schemes.
2. We determine the criteria we will use to evaluate our proposed schemes. The primary criteria is the park user's enjoyment, which we represent by a utility function.
3. We present the relevant data on Topoland. This data is estimated based on large-scale amusement parks.
4. We state assumptions for our park users. These assumptions are based on and justified by our own experiences.
5. We construct and analyze a model to evaluate our QuickPass schemes.
6. We implement a computer simulation to examine our proposed QuickPass schemes.
7. We estimate these computer simulations to ensure that they make sense.
8. Based on the results of the computer simulation we give strong recommendations on implementing QuickPass schemes.

3 QuickPass Schemes

To narrow the large number of QuickPass schemes we could consider, we first specify necessary conditions for a QuickPass scheme and eliminate possible schemes that we consider to be ineffective. **We choose QuickPass schemes with the following properties:**

1. **Users can only hold one QuickPass at a time.** If not, early-arrival park users have a huge advantage on late-arrival users.

2. **Cancelled QuickPass times are not reentered into the system for others to use.** Sequential distribution of QuickPass times eliminates the possibility that given two QuickPass users, the second user to request a QuickPass will not receive an earlier return time than the first which would be clearly unfair.

3. **A limited number of QuickPasses should be issued per time interval of the day.** This excludes what we refer to as the "Line Loan QuickPass" scheme. The "Line Loan QuickPass" scheme never rejects an attempt to obtain a QuickPass and merely defers a park user to a later time. This effectively decreases line lengths for a short time, but then QuickPass users accumulate later in the day and end up waiting in a QuickPass line as long as the normal line.

These potential problems and the problems with Topoland's QuickPass system serve as a reminder that QuickPass schemes must be given much consideration before proposal. The following subsections describe our candidate schemes, which we will refer to as *No QuickPass Scheme*, *Fixed-Rate Scheme*, and *Single Line Scheme*.

3.1 No QuickPass Scheme

This scheme will serve as a comparison for evaluating other schemes. Although the "Non-Scheme" should always be tested in models, it is especially important in this problem because we are optimizing user "enjoyment"- a quantity which can only be thought of in relative terms. Without the No QuickPass scheme, "enjoyment" quantities have little meaning.

3.2 Fixed-Rate Scheme

Each park ride has the capacity to entertain a fixed rate of people. This scheme reserves a fixed proportion ϕ, of the ride capacity for QuickPass users. Thus, QuickPass users are sequentially issued return times based only on ϕ and on the number of previously queued QuickPass users. Upon returning, QuickPass users enter a different line than normal users and are allowed to enter into the ride at the rate ϕ. We do not propose a value for the rate ϕ at this time, for we will optimize it in our model later. QuickPass users cannot be expected to return at an exact time, so we must provide a time window for which they are permitted to return. This time window is addressed in Section 6.6.

Hypothesized Advantages and Disadvantages

Advantages of this scheme include easy implementation and administration. A fixed merging rate ϕ, is much easier for park attendants to use than a merging rate that varies with time or line length.

Disadvantages of this scheme include the necessity for two lines: one for normal users and one for QuickPass users. Topoland already has already installed the necessary facilities because they already have a QuickPass system in place, but this scheme requires more park attendants than the No QuickPass Scheme.

3.3 Single Line Scheme

In this scheme, QuickPass return times are issued according the following equation:

$$(1) \quad QuickPassReturnTime \equiv NormalLineTime + QuickPassLineTime$$

Here, *NormalLineTime* is the expected difference in the time between entering the line and loading the ride. *QuickPassLineTime* is the expected QuickPass return time for a ride. In this scheme, the normal line and the QuickPass line are effectively the same. Thus, users will more frequently request a QuickPass than stand in the normal line. QuickPass users returning for the ride are loaded before normal users in this scheme.

Hypothesized Advantages and Disadvantages

Advantages of this scheme include easy implementation and administration. Since the opportunity cost for waiting for the ride decreases, there are more QuickPass users in this scheme. Thus, opportunity is provided for more park users to take advantage of the QuickPass. However, this also lengthens the QuickPass return times. Here again, the time window for which QuickPass users are allowed to return must be considered. This issue is addressed in Section 6.6.

4 Our Model: Evaluating Schemes

Our schemes must be evaluated with certain objectives in mind. Here, in order of decreasing importance, we list the two primary considerations when constructing our model. Our scheme should:

1. Increase park user's enjoyment
2. Give all park users an equal opportunity.

4.1 The Model

To evaluate a person's enjoyment in Topoland, we assume that their enjoyment depends only on the following:

- the quantity of rides the user enjoys
- the quality of rides the user enjoys
- the amount of time spent in line for these rides

Assuming such, we assign a *user utility function*, ΔU, that measures the change in enjoyment of an individual user. The users in our model will also rely on this function to choose a ride or become idle.

$$(2) \qquad \Delta U_i \equiv \begin{cases} RideScore - LineTime \times User'sPatience & \text{user rides} \\ -User'sIdleConstant \times IdleTime & \text{user idling} \end{cases}$$

Here, *RideScore* is a quantity each ride in the park is assigned based on its popularity. This process is described in Section 4.2 and also the rides of Topoland are assigned scores. *LineTime* is the expected wait time for a ride's line. *User'sPatience* and *User'sIdleConstant* are both quantities assigned to each individual user. They will be discussed in Section 4.3. *IdleTime* is the time in which a user is idle, that is, not in a line and not on a ride.

Then, for each individual user, the sum over the course of a day, $U = \sum_{day} \Delta U$, represents the enjoyment he or she had that day. The mean U value represents the average enjoyment of park users. The standard deviation in U values is the typical variation a user had from the mean value. Thus the standard deviation is a measure of a scheme's fairness. A scheme with a low standard deviation gives each park user an equal opportunity.

4.2 Details of Topoland

For a given amusement park, we assign each park ride a score (0-100) based on its popularity. The larger the score, the longer an average individual is willing to wait for that ride. For Topoland, we ranked the park rides as listed in Table 1. The *RideTime* and Passenger Capacities are estimated based on large-scale amusement parks. [7, 3]

4.3 Details of the Park Users

Before we can measure the enjoyment of our park users,**we assume the following:**

- **Park users have a uniform preference for each of the rides.** This assumption makes the scoring of the park rides well-defined.

Table 1: Scores of the Rides in Topoland

Score	Ride	Ride Time	Capacity
100	Möbius Madness Roller Coaster	10 min.	90
90	House-o-Hausdorff	10 min.	90
65	Homotopic Horror Roller Coaster	10 min.	80
50	Torus Float	10 min.	80
40	Krazy-Kliene-Kar	10 min.	70
10-20	20 Carnival-Type Rides	10 min.	30-50
10-20	15 Children's Rides	10 min.	20-40
10-20	5 Longer Rides	15-25 min.	80-120

- **Park users are aware of the line length for each of their choices.** This assumption is realistic in parks such as Topoland that have screens throughout the park which display the line length for each ride.
- **Park users attempt to maximize the quantity and quality of their ride time and minimize their time spent in line.**
- **The occupancy of the park is a known distribution with respect to the time of the day.** This can be realistically estimated based on past park occupancy distributions.
- **We assume that each user has a well-defined quantity that we refer to as his or her *User'sPatience*.** This constant defines a linear relationship between the popularity of a ride and the time in which a user is willing to wait for it.
- **We assume each user has a well-defined quantity that we refer to as the *User's Idle Constant*.** This constant defines the length of time in which the rider will no longer wait in line.

The two constants mentioned above - *User'sPatience* and *User's Idle Constant* depend upon the individual, but the average can be calculated in the following manner. Based on data collected in Topoland, we determined that the average park user will wait for 120 minutes in line for the Möbius Madness Roller Coaster and enjoy their time spent. Thus we calculate *User'sPatience* by the equation:

$$\Delta U_i = 0 = (100pts.) - (120minutes) \times (User'sPatience) \tag{3}$$

Thus we obtain an average value of *User'sPatience*=.83.

Also from data collected in Topoland, we determind that the average park user will not join the line for Möbius Madness Roller Coaster if it is more than 150 minutes. With this value, we estimate the average value of *User's Idle Constant* to be:

$$\Delta U_i = 0 = (100pts.) - (150minutes) \times (User'sPatience) \tag{4}$$

Thus we obtain an average value of *User's Idle Constant*=.2.

4.4 Discussion of Our Model

In our model, if each of the users has an equal opportunity and each maximizes their utility function, we expect all users to have roughly the same utility function value and for each to be greater than or equal to the utility value for an idle park user (see Table 2):

$$\Delta U_1 = \Delta U_2 = \cdots = \Delta U_n \geq \Delta U_c \tag{5}$$

The inequality will hold be an equality if and only if there are idle users.

One of the strengths of this model is its capability to adapt to different parks and to various occupancy distributions. To investigate an additional park, say *Algebraland*, an administrator only needs to assign each ride in the park a rating. Also it is quite possible that a ride's popularity might change over the course of a day and this variation can also be accommodated. For example, the *Torus Float*, being a water ride, would be much more popular during the hot hours of the day.

Our model can also be adapted to monitor other useful data. For instance, a park manager could decide to optimize the amount of time park users spend doing a specific activity. We do this by treating say, the *Tori Bakery* as a ride in our model. Adjustments in QuickPass schemes could then be made to increase the number of people buying higher genus donuts in the Tori Bakery.

The major weakness of our model is due to the large number of assumptions we have made. In reality, park usrs do not have a uniform preference for each of the rides. However, as user preferences scatter, our model will underestimate the average utility values. This can be consempated for by underestimating the park user population. Another weakness in our model is the assumption that users have these linear constants- *User'sPatience* and *User's Idle Constant*. There is little reason for these values to be linear on a large range of time.

5 Estimations

5.1 Idle Users Matter

Here we show by approximation that the effectiveness of a QuickPass scheme can be calculated by the number of additional idle users. Because of the large number of variables, we have compiled them in Table 2 for the reader's convenience.

For each user, we have

$$\frac{dU}{dt} = \begin{cases} RideScore/RideTime & \text{Riding} \\ -p & \text{In Line} \\ -c & \text{Idling} \end{cases} \tag{6}$$

Table 2: Definition of Several Variables Used

Variable Name	Definition
p	Average User's Patience
c	Average User's Idle Constant
R_i	User on a Ride
Q_i	User in Line
I	Total Number of Idle Users in the Park
T	Total Number of Users in the Park
ΔU_i	The Expected Utility Difference
ΔU_c	The Idle Utility Difference

When Equation 6 is summed over all park users, the utility change in the whole park is obtained:

(7)

$$\begin{aligned}\sum_{user}\frac{dU}{dt} &= \sum_{users-on-ride}\frac{RideScore}{RideTime}+\sum_{users-in-line}-p+\sum_{users-idle}-c\\ &= \sum_{attraction}\left(R_i\times\frac{RideScore}{RideTime}\right)-\sum_{attraction}Q_i\times p-I\times c\\ &= \sum_{attraction}\left(R_i\times\frac{RideScore}{RideTime}\right)-\left(T-\sum_{attraction}R_i-I\right)\times p-I\times c.\end{aligned}$$

We assume that almost every seat in each major attraction is taken, and write $\sum Q_i$ in terms of T, R_i, and I. We know that

$$T=\sum_{attraction}R_i+\sum_{attraction}Q_i+I \tag{8}$$

(9)

$$\sum_{users}\frac{dU}{dt}\approx\sum_{attraction}\left(R_i\times\frac{RideScore}{RideTime}\right)-\left(T-\sum_{attraction}R_i\right)\times p+I\times(p-c)$$

Since the first term only involves the attraction properties, and the second term doesn't depend on the choice of QuickPass scheme, $I\times(p-c)$ is the only term that depends on the QuickPass scheme. Conceptually, It can be realized as that the QuickPass system removes people from the normal waiting line, so those park users now suffer c rather than p.

More precisely, the coefficient $(p-c)$ is exactly the difference between standing in line and idle, which makes a perfect sense with the above reasoning. This effect is positive since p is defined to be greater than c. Thus, it is clear that the effect of QuickPass comes from the increase of idle users.

5.2 QuickPasses are Effective

Here we show that the effectiveness of implementing a QuickPass scheme on a ride is proportional to the popularity of the ride. When there are some idle park users, we know that $\Delta U_i = \Delta U_c \approx 0$, so

$$\Delta U_i = RideScore - LineTime \times p \approx 0 \tag{10}$$

$$LineTime \approx RideScore/p \tag{11}$$

Now suppose we issue QuickPasses at a constant rate at the ith attraction,

$$Q_i \approx (RideOccupancy - QuickPassPerRide) \times LineTime/RideTime \tag{12}$$

$$Q_i \approx \frac{RideOccupancy \times RideScore}{RideTime \times p} - QuickPassPerMinute \times \frac{RideScore}{p} \tag{13}$$

$$R_i \approx RideOccupancy \tag{14}$$

$$I = T - \sum_{attraction} (Q_i + R_i) \tag{15}$$

$$\begin{aligned} I =& T - \sum_{attraction} \left(RideOccupancy - \frac{RideOccupancy \times RideScore}{RideTime \times p} \right) \\ &+ \sum_{attraction} \left(QuickPassPerMinute \times \frac{RideScore}{p} \right) \end{aligned} \tag{16}$$

Since only the last term involves the QuickPass scheme, the number of idle users has a positive linear relationship with the amount of QuickPass issued at each attraction, while the coefficient is propotional to its *RideScore*.

We can realize this effect as that when a virtual queue comes into play, it comparatively narrows down the corresponding normal queue and push people out from it, and therefore increases the number of idle users.

In addition, we know from Section 5.1 the utility gain by QuickPass is determined by $I \times (p - c)$. This shows that when there are idle users, issuing QuickPasses on popular attractions has a larger effect on the change in user utility than low scored ones.

5.3 Upper Bound on Waiting Time

In all cases we have $\Delta U_i \geq \Delta U_c \approx 0$, thus

$$\Delta U_i = RideScore - LineTime \times p \geq 0 \qquad (17)$$
$$LineTime \leq RideScore/p$$

This gives an approximate upper bound on $LineTime$.

Similar to the effect in Section 5.2, when there are no idle users, opening a vitural queue increases the overall waiting time in the normal lines. When there are no idle users, if we increase the ratio of QuickPass users, the waiting time will increase until it reaches this upper bound. After it reaches this upper bound, it remains constant while idle users appear and affect utility gains. In brief, the advantage of the QuickPass system is insignificant when we only issue a few passes, but it becomes significant as more idle users appear.

6 Park Simulation

6.1 Assumptions

Some of the interactions between real park users and real parks are too complicated to simulate in a computer. We therefore make the following assumptions to simplify the problem:

- **The park distribution is simple.** Topoland opens for 12 hours a day. The 6 hours in the middle are considered to be busy hours. Park users arrive and leave sequentially in the first and last three hours, rather than rush in and out simultaneously. The park occupancy distribution assumed is depicted in Figure 3.
- **Park users act in groups of 10.** This reduces the computation time by a great factor, and the effect of the clustering tends to average out when the number of park users is large.
- **Park users spend minimal time walking and always use their QuickPasses.**
- **Each park user has the same User's Patience and User's Idle Constant** These are set to the average values obtained in Section 4.3.

6.2 Formulation

We construct the simulation as follows:

- Each attraction in the park is represented by a real queue and a virtual queue. The real queue holds the park users who wait in the normal line, while the virtual queue tracks the park users who use QuickPasses.

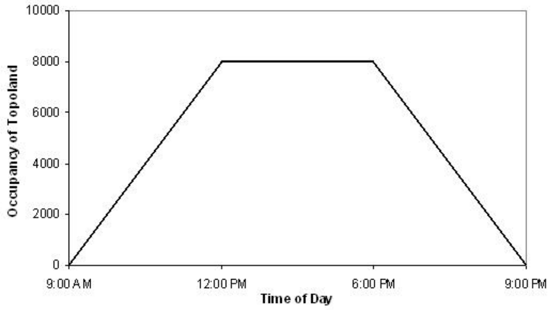

Figure 3: Park Occupancy Over the Course of One Day

- The park configuration consists of the set of attractions and the position of the park users.
- We keep track of the changes in the park in discrete time steps of 10 minutes. At each step, the status of each attraction is updated, and idle users choose their next move.
- The utility gains (or losses) among all the users in the park at each time step are collected and summed to give the total utility gain for a day.

6.3 Stability Check

In order to guarantee that the result of simulation doesn't shift too much when the inputs change slightly, we make the following variations on the inputs and monitor the output of the simulation:

- We vary the occupancy of the park by $\pm$ 200 people. The average utility varies less than 2%.
- We slightly change the score list of the attraction set by 1% to 4%, but keep the ratio among each ride score. The output shows no significant difference.
- We change the group size of users to 1, 2, and 5. The result only changes by tiny amounts.

6.4 Schemes and Park Configuration

We considered the following variations for our proposed scheme:

- Nine QuickPass schemes: No-Pass Scheme, 7 variations of Fixed Rate Scheme, and Single Line Scheme. Details of the 7 different types of Fixed Rate Schemes are shown in Figure 4.

Scheme 1	High	Med.	Low
Morning	50%	25%	0%
Mid-day	75%	50%	0%
Night	50%	25%	0%

Scheme 2	High	Med.	Low
Morning	25%	0%	0%
Mid-day	50%	25%	0%
Night	75%	50%	25%

Scheme 3	High	Med.	Low
Morning	75%	50%	25%
Mid-day	50%	25%	0%
Night	25%	0%	0%

Scheme 4	High	Med.	Low
Morning	50%	0%	0%
Mid-day	50%	0%	0%
Night	50%	0%	0%

Scheme 5	High	Med.	Low
Morning	75%	0%	0%
Mid-day	75%	0%	0%
Night	75%	0%	0%

Scheme 6	High	Med.	Low
Morning	75%	50%	0%
Mid-day	75%	50%	0%
Night	75%	50%	0%

Scheme 7	High	Med.	Low
Morning	75%	75%	0%
Mid-day	75%	75%	0%
Night	75%	75%	0%

Figure 4: Variations of Fixed-Rate Schemes Simulated

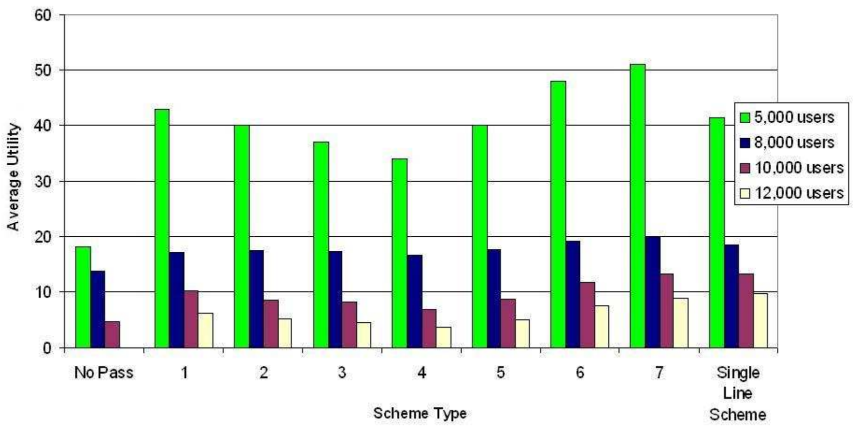

Figure 5: Average Utility for Various Schemes and Occupancies

For the park:

- Four values for park attendance: 5000, 8000, 10000, and 12000. These reflect the different crowd conditions between holidays and weekdays.
- The attraction set includes 2 coasters with high scores, 3 coasters with medium scores, and 40 rides with low scores (see Table 1).

6.5 Results

Results from our simulations are presented in Figure 5. Different bar shades correspond to the park at different occupancy levels, and the variations of the schemes are defined in Section 6.4. Apparent from Figure 5, it is clear that different schemes perform well under different conditions. Clearly, Type 7 of

Table 3: How Variations in the Time window Effect the Average Waiting Time

Time Window	Resulting average waiting time
30	5.5
45	5.6
60	5.7
75	5.8
90	6.1
120	6.2

the Fixed-Rate Scheme performs best in the rage of values $< 11,000$. The Single Line Scheme performs best in the range $> 11,000$.

6.6 Time Windows

When users recieve a QuickPass, they are assigned a return time. Since they cannot be expected to return at an exact time, a time window in which they are permitted to return is assigned. In this section we investigate how the average line length is dependent on the length of the time window. We use a program to calculate the expected waiting time for various time windows. Taking the Möbius Madness Roller Coaster as an example, the time per ride is ten minutes and the occupancy per ride is 90 users. In Table 3, the resulting average waiting times are computed for various time windows.

Clearly, the resulting average waiting time is not significantly affected by the time window assigned. We suggest a one hour time window, because smaller time windows may cause users inconvenience, while larger time windows may still lead to a comparatively longer waiting time.

7 Discussion

From the results we can see that several of the schemes compete for maximizing the utility function over different ranges in park occupancy. Therefore we could proposed multiple schemes that are implemented depending on the park occupancy. It will be difficult to implement if we have too many candidates for schemes, it will make the QuickPas system unnecissarily complicated. Also, it is necessary for administration purposes not to change schemes mid-day. This can be prevented if the crowd size is accurately predicted each day. It is clear that implementing a small number of schemes maximizes the user function over the largest population distribution and it will not cause too much confusion for park administrators.

A disadvantage in all of these schemes is that they can defer QuickPass users for too long. For instance, the Topological Amusement Park has two main attractions: the Möbius Madness Roller Coaster and the House-o-Hausdorff Ride. Suppose that the line for the Möbius Madness Roller Coaster is x minutes long and also that the House-o-Hausdorff Ride routinely distributes QuickPass tickets that defer riders for some time greater than x. This creates a situation in which park users who arrive early are able to take advantage of the House-o-Hausdorff Ride QuickPass and while waiting for their time window, ride on the Möbius Madness Roller Coaster. As one can easily see, this is not fair to users who arrive to the park later in the day. In our model this would be refelcted in the standard deviation of user utility functions, but this was not investigated in our simulation.

This equal opportunity for all park users problem has long-term consequences on Topoland. If people feel that the system is unfair, they will be unwilling to return to the park. Since our model evaluates only short-term effects, this problem should be considered simultaneously.

Thus, we provide a few solutions to prevent this practice from occurring:

1. **Active Ride Restriction Program** This program actively prevents park users from using other major attractions while they hold a QuickPass. This would require that Park Tickets were checked upon entry into a normal line. This is an added expense but would prevent this pitfall.

2. **Passive Ride Restriction Program** This program attempts to deter park users from using other major attractions while they hold a QuickPass. This is implemented by offering *coupons* on the QuickPasses to encourage QuickPass holders to shop in one of the many *Topoland Gift Shops* or enjoy a meal from the *Disjoint Dinner*. This not only reduces the problem but also increases the revenues of Topoland. However, this does not actively prevent QuickPass holders from joining another major line in the park.

3. **Compromise Restriction Program** This program is not active but is more aggressive than the Passive Restriction Program. In this program, QuickPass holders are required to activate their QuickPasses at one of the restaurants or shops in the park. This is tempting trip is bound to prevent some of the QuickPass holders from returning in time to stand in another major line. Although this program is likely to make park users upset.

We want to ensure that our proposed QuickPass scheme does not have the same problems as the Topoland's current scheme. Currently, users in Topoland may insert their ticket into the Kiosk to receive a time. Just a short time later, a user will insert their ticket into the Kiosk and receive an earlier time. This disorder of return times is possibly because some users cancel their QuickPasses, creating holes in the virtual queue. The system must then redistribute these QuickPasses to new requests. Since we do not allow for this to happen, our scheme will not have this problem.

Also, the lines for people with Quickpasses are nearly as long and slow as the regular lines in the current Topoland scheme. Because we control the rate at which tickets are distributed in our scheme, park users experience almost no line as shown in Table 3.

The cost of implementation is also an aspect of the evaluated schemes that should be considered. Since the infrastructure is already in place for a QuickPass scheme, these costs will be low. Thus, they do not significantly effect our proposed schemes.

8 Conclusions and Recommendations

8.1 Conclusions

- From the estimation in section 4, we derive some interesting conclusions about the QuickPass system. First of all, the total advantage gained from the system merely depends on the increased number of idle park users, because QuickPass actually transfers users who were originally stuck in line to idle status.
- Secondly, the number of idle users has linear relationship with the amount of QuickPass issued at each attraction, while the coefficient is proportional to its *RideScore*. This simply shows that QuickPass works better with popular rides.
- We also conclude that there is an upper bound on the waiting time at each attraction, given by $RideScore/w$. The QuickPass system shows its great power when the amount of QuickPass issued is large enough so that there idle users appear in the park.
- There is no single scheme that is best over the entire range of park populations.

8.2 Recommendations

- **To maximize park users' enjoyment:** If the occupancy of Topoland is lower than $11,000$ people, we strongly recommend Type 7 of Fixed Rate Scheme. However, when the park occupancy exceeds $11,000$ people, we think the Single Line Scheme is the best choice. In both cases, the QuickPass system can be easily implemented without too much marginal cost.
- **To give park users as equal opportunities as possible:** Among the three different strategies, we suggest the Active Ride Restriction Program. This should make the QuickPass system more balanced, and create more revenue in long term business for Topoland.

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