Analysis of Keep-Right-Except-To-Pass Rule

Control Number 29224

February 10, 2014

Abstract

In this paper, we present a computerized model by applying the method of Cellular Automaton to simulate a series of cars' actions under the rule restriction. With the theory of probability that any given object move to its neighborhood with certain probability based on speed, risk coefficient, and etc, we calculate all possible steps for each object and generate a simulation which performance the traffic flow under different situations not limiting on light and heavy traffic. In order to analyze the influence of the Keep-Right-Except-To-Pass Rule (KRETPR) on the tradeoff between traffic flow and safety issue regarding to modern freeway traffic, a new model based on the microscopic scale cellular automaton model is proposed. Traditional Cellular Automaton model with single-row lattice is expanded into a multi-row two-dimensional lattice as the simulation of multi-lane freeway, with number pairs describing the vehicular status and inter-vehicular interaction.

Contents

1 Introduction **3**
1.1 The Common Rule 3
1.2 Goal Of The Model 3
1.3 Existing Models For The Problem 4
1.3.1 Microscopic Scale Models 4
1.3.2 Macroscopic Scale Models 4
1.3.3 Mesoscopic Scale Models 5
1.3.4 Our Approach 6

2 Approach **7**
2.1 Multi-Lane Freeway Model 7
2.2 Vehicle Speed Model 7
2.3 Cell State Update Rules 8
2.3.1 Terminology 8
2.3.2 Order of Calculation and Update 9
2.3.3 Correspondence to KRETPR 9
2.3.4 Correspondence of Speed to Normal Distribution 9
2.3.5 Reasonable Speed Change 9
2.3.6 Risk 10
2.3.7 Summary 12
2.4 Flow and Safety 12

3 Result **12**

4 References **15**

1 Introduction

Tradeoff between traffic flow, i.e. efficiency, and safety of traveling by car has been a main concern in the field of traffic regulation with the preference of driving in short and medium distance. Different rules are adopted to cope with such problem in case of multi-lane freeway in countries of different driving style. The effectiveness of one common rule will be analyzed through building mathematical model.

1.1 The Common Rule

For most countries where vehicles are driving on the right of the road, the common rule for multi-lane freeway that is the Keep-Right-Except-To-Pass Rule (KRETPR), where drivers are required to drive in the right-most lane. The rule is ought to be obeyed unless vehicles are passing over another vehicle, in which case they move one lane to the left, pass, and return to their former travel lane. This overtaking process relies on human judgment for compliance.

1.2 Goal Of The Model

The first goal of the model is to analyze the performance of the KRETPR under different conditions by comparing the traffic flow and the risk of car accidents. Possible rules better than KRETPR will be provided as well. Only common conditions with an observable potential influence on traffic flow and safety are considered in the analysis, which are listed as follow:

(a) The effectiveness of KRETPR in promoting better traffic flow under light and heavy traffic conditions and the magnitude to which they are affected by this rule.

(b) The effectiveness of KRETPR under different sets of maximum and minimum speed limits and compare the currently adopted speed limit with the most appropriate ones from calculation.

(c) The effectiveness of KRETPR under different weather conditions like rain, snow, ice, and fog as for their influence on the fraction factor between tire and road surface as well as visibility. Weather conditions with different magnitude are also considered.

The model is also designed to analyze the possibility of adopting a corresponding rule, i.e. Keep-Left-Except-To-Pass Rule (KLETPR), in countries where vehicles are driving on the left. The KLETPR is only different from KRETPR in a simple change of orientation of the preferred lane. Other possible factors such as vehicular interior arrangement and the influence of the neighboring lanes, which are in the opposite direction, will be discussed.

The third goal of the modern is to adjust the existing model by introducing new parameters in order to cope with the situation where vehicle transportation on the same roadway was fully under the control of an intelligent system through either part of the road network or the imbedded structure in the design of all vehicles using the roadway. The effectiveness of the adjusted model will be analyzed under the aforesaid conditions that would influence the traffic flow and safety of vehicular transportation.

1.3 Existing Models For The Problem

The key of the model is the simulation of the freeway?s traffic flow in order to predict the vehicular speed and density in any future moments. The existing approach can be divided into three categories with respect to the scale of observation: microscopic scale, macroscopic scale, and mesoscopic scale.

1.3.1 Microscopic Scale Models

The often-used microscopic scale models are the Nagel-Shreckenberg (NS) model and Fukui-Ishibashi (FI) model, which are based on cellular automaton (CA) and every vehicle is treated as individual. The freeway in these models is treated as a one-dimensional lattice and the vehicular motion is treated as the hopping processes within the lattice [2]. Classes of vehicle m is also considered for their difference of maximum velocity v_m. The key of CA model is the treatment of individual lattice as a number pair with two variables, the coordination of position j and the coordination of time t. Therefore, the configuration of vehicles of class m on individual lattice can be reproduced by a variable $\sigma_m(j,t)$, where $\sigma_m(j,t)$ possesses the value of 1 if any vehicle of class m occupies the site and possesses 0 for the rest of situations. Therefore, the lattice configuration can be represented by the sum of vehicles of different classes

$$\sum_{m=1}^{M} \sigma_m(j,t)$$

with great simplicity. Another feature of the CA model is the simulation of vehicular advancement in the one-dimensional lattice through the defining the slowdown factor $\exp[-W(j,\sigma(j,t))]$ as the influence of other vehicles with respect to distances.

All vehicles' states can be deduced via iteration of the ordinary differential equation (ODE) of the position function $\sigma_m(j,t)$ and the expectation of transition probability E of neighboring vehicles.

$$\frac{d}{dt}E\sigma_m(j,t) = EG\sigma_m(j,t)$$

where G is the generator of Markov process (σ_m, σ). However, none of the existing CA models take into consideration the overtaking process, which involves the cellular hopping between one-dimensional lattices.

1.3.2 Macroscopic Scale Models

The most commonly adopted macroscopic scale model is the Lighthill-Whitham (LW) model of fluid dynamics simulation, where traffic flow is literally considered as a flowing liquid and the theory of fluid dynamics is applied to analyze the speed u, density k, and their multiplication, i.e. the flux q of the traffic flow.

$$q = ku$$

This model deals with the overtaking process effectively by solving partial differential equations (PDE) system based on the hypothesis that the sum of increase of traffic flow density in unit time $\frac{\partial k}{\partial t}$ and the change of traffic flux in unit length $\frac{\partial q}{\partial x}$ is zero. [4]

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0$$

Overtaking, in this case, is simulated as the viscosity coefficient μ as a clear reference to the hydrodynamics, where τ is defined as a value that is proportional to the shear velocity and contacting area, which is

$$\tau = \lambda A \frac{\partial u}{\partial y}$$

In the case of improved LW model, the viscosity coefficient of traffic flow is defined as

$$Z = f[\mu \Delta v \frac{h}{h - d_{min}}]$$

where Δv is the speed difference, and $d_m in$ is the safety distance. Combining the PDE of traffic flow and flow density, the adjustment factor for overtaking, and the classical velocity-acceleration ODE in the Newton's Second Law of Motion, which is

$$\frac{du}{dt} = \frac{1}{T}[u_e(k) - u(x,t)] + \mu$$

where u stands for vehicle speed and T stands for the necessary deceleration time at the safety distance. Consequently, it is possible of applying to the KRETPR with an additional coefficient of perpendicular movement, yet there might be bigger deviation from reality due to its limited modeling power.

1.3.3 Mesoscopic Scale Models

In a common mesoscopic scale models, the Anisotropic Mesoscopic Scale (AMS) model, the vehicular speed response is represented in a macroscopic manner while the influence of vehicles in front of individual cars in the speed influencing region (SIR) are considered when analyzing their speed in a similar way in the microscopic scale model. The calculation is based on the relationship between traffic density k and speed of i th vehicle in the SIR

$$v_i^t = p(k_i^{t-1})$$

where $p : k \to v$ is the non-increasing speed-density relationship function [3]. The advantage of AMS is the simplification of iteration of every single vehicle units in the CA model while keeping a relative comprehensive concern of various factors that influence traffic flow and safety.

1.3.4 Our Approach

Our model is mainly based on the microscopic scale model of CA with necessary adjustments to fit into the KREPTR, where speed of different lanes is influenced by all driver's tendency of driving on the rightmost lane unless overtaking other vehicle. Rules are introduced to differentiate inter-lane and intra-lane position change with an incorporation of probability theory through separate consideration of vehicle and position. Details of the model will be discussed in following paragraphs.

2 Approach

2.1 Multi-Lane Freeway Model

The foundation of our model is an expanded cellular automaton model with a two-dimensional lattice grid map as the simulation of multi-lane freeway, where status of individual vehicle is iterated in order to maintain the accuracy and comprehensiveness while simulating the freeway driving in reality. The adoption of two-dimensional lattice grid map is also necessary as for simulating the overtaking process, where neighboring lanes are involved, as a basis of introducing the KRETPR into our model. The two-dimensional grid map used in our model is an $m * n$ lattice, where m is the number of rows (lanes) and n is the length of each row (lane). The states of cell (i, j) in the $m * n$ grid is defined via two rules:

1. the cell possess a value of -1 if there is no car occupying the position (i, j) at this moment;
2. the cell can possess any value except -1 if it is currently occupied by the cth car;

To analyze the situation of overtaking, the neighborhood of a certain cell should be considered so as to avoid potential collision when changing lanes. The neighborhood of cell (i, j) is defined as (i', j') where:

1. $1 \leq i' \leq m$ and $1 \leq j' \leq n$;
2. there exists a car c in the cell (i', j') in the current situation;
3. it is possible for c to go to cell (i, j) at the next iteration.

This is the basic model that we use to simulate the behavior of multi-lane highway. Additionally, there should be one or more rule(s) adopted when updating the cells' states in each iteration regarding kinematics among individual cars and their safety issue. The rules will be discussed in detail in section **2.3**.

2.2 Vehicle Speed Model

The speed of individual vehicle is crucial during the process of simulation, not only for analyzing overall traffic conditions, but for judging the possibility of lane-change in the overpassing situations. Numerous data regarding existing regulation of freeway vehicle speed can be found in publication and reports from the government as well as NGO, yet none of them show the concern of KRETPR. Therefore, we chose to produce a general model for all potential cases before applying any of the data into our model considering the variation of difference of road situations and specific rule details in different areas. Data in our model is generated through mechanical simulation for further comparison with the following method. Suppose the speeds of vehicles on a freeway $speed_c$ obeys Normal Distribution, which is the simplest and most realistic distribution to describe the randomness of speed of different cars, where number of cars with an extremely high and extremely low speed is small considering the speed limit of the freeway. So, the speed of individual car c on the freeway would be:

$$speed_c = \mu + \sigma \times \mathcal{N}(0, 1)$$

where μ is the pre-defined average speeds on the highway and σ is the pre-defined standard deviation of speeds. Therefore, $\mu + 4 \times \sigma$ and $\mu - 4 \times \sigma$ are the highest and lowest speeds on the highway respectively.

In addition, we could also simulate different road conditions mentioned in the Introduction paragraphs, like weather and potential influence of traffic accident, by adjusting the value of μ and σ. For example, μ is smaller in sunny days than in storm days and σ is smaller on all-good highway than the highway with several defective segments.

2.3 Cell State Update Rules

How to update cell states for all individual cars in a comprehensive while simple way is our next major concern after the generation of the two-dimensional lattice. At each iteration of CA, we need to update cells' states reasonably. The confinement of real situation is summarized into the following rules:

1. all updates must be in accordance with the KRETPR;
2. the speed of vehicle at the iteration must correspond to the normal distribution stated above;
3. acceleration and deceleration of any car must be reasonable so that a car whose speed changes significantly never exists;
4. drivers are all intelligent, and they will choose a path where the potential risk they undergo is as low as possible.

We add the fourth rule to keep consistence with the given conditions of the problem as well as for simplification. For each car, we calculate the probability that the car c will go from cell (i, j) to (i', j') in next iteration, then we move the car to its most likely cell accordingly. For car c, if its most likely target cell is conflicted with the cell that has been occupied by another car, it should go to its next most likely target. Two cells are conflicted may because they are actually the same cell, or the distance between these two cell is less than the safety distance.

In order to simplify the computation, instead of iterating every cell (i, j) and calculating the probability that its neighbors will go to itself, we iterate every car c and calculate the probability for each cell (i, j) that c might go.

2.3.1 Terminology

1. $P_{car}(c, i, j)$: the probability that car c will go to (i, j) in the next iteration;
2. $P_{loc}(c, i, j)$: the probability that there will exist car at (i, j) in the next iteration before $P_{car}(c, i, j)$ is calculated.

2.3.2 Order of Calculation and Update

In the real world, when a driver decides to turn, accelerate, or change lane, he always looks at his front car, and makes decision mainly according to the movement of his front car. So intuitively, in our model, front cars have priority to calculate probabilities and choose its most likely cell as its target. Besides, since we have to follow KRETPR rule, so we give the same priority to the car in the left lane.

2.3.3 Correspondence to KRETPR

In each iteration, a car has three options in directions choices: 1. moves one-step left; 2. remains in the same lane; 3. moves one-step right. According to KRETPR:

1. in option 1, the speed of the car has to be faster than its front car. So when we consider the potential target in the left lane of car, we only consider cells to which car will have larger speed than its front car;
2. $priority(3) > priority(2) > priority(1)$. After we calculate all the probabilities of car's potential target location, we factor the cells in the right lane by 0.45, the cells in the same lane by 0.35, and the cells in the left lane by 0.2.

2.3.4 Correspondence of Speed to Normal Distribution

For each potential target cell (i', j') for car c, we calculate the speed v that c needs to have to reach from its current cell to (i', j'). As we mentioned above, the speeds fit normal distribution with parameter μ and σ. The we calculate the probability that v exists under such normal distribution by probability distribution function of normal distribution:

$$P_v(v) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(v-\mu)^2}{2\sigma^2}}$$

Intuitively, $P_{car}(c,i,j)$ increases when $P_v(v)$ increases.

2.3.5 Reasonable Speed Change

On highway, it is very dangerous to accelerate or decelerate instantly. So we also consider the speed of change in the next iteration as a part of $P_{car}(c,i,j)$. The larger speed change is, the less the probability that the car c goes to (i,j) in next iteration is.

2.3.6 Risk

Suppose all the drivers are intelligent and all of them care about their own safety. As a result, every time when a driver makes a decision, he will consider the potential risks caused by this decision. In this problem, the potential risks mainly come from the relationship between actual distance between two cars and their safety distance. Specifically, if actual distance between two cars is greater than the safety distance, when the potential risk is 0, or infinitely small. When the actual distance between two cars is smaller than the safety distance, then the potential risk increases when the actual distance decreases, and the potential risk will increase exponentially.

Assume car c follows car c' on freeway, and speed of c is v. Then we define the safety distance as the distance that c needs to completely stop from its original speed. Then we could have

$$v^2 - 0^2 = 2ax$$

[1]

Where 0 is the speed when c is completely stopped; a is acceleration (related to car's model); and x is the distance that c needs to completely stop from its original speed, which is safety distance. Then we could conclude

$$x \propto v^2$$

Then risk potential is defined as

$$risk_{potential} = e^{-r\frac{x}{l}} = e^{-r\frac{v^2}{l}}$$

Where r is a constant coefficient, and l is the actual distance between c and c'.

Since the driver of c will never know which cell in front of him will be occupied until the next iteration, so he has to approximate the risk potential caused by movement from cell (i,j) to cell (i',j').

There are two types of risk potential in this case. The first one is risk potential between c to its front car $risk_{front}$, the second one is risk potential between c to its back car $risk_{back}$. Here are the methods to calculate them.

1. Assume the potential target cell of car c is (i, j) and we are about to calculate the potential risk between c and car c' that might appear at cell (i, j') and $j' \geq j$. Then we have

$$risk_{potential}(c, c') = \prod_{j+1}^{j'-1} P_{loc}(c, i, j) \times e^{-r\frac{v^2}{l}}$$

Where v is the speed of c when it moves from its original cell to (i, j). Since l is distance between c and c',

$$l = j' - j$$

Then we get

$$risk_{potential}(c, c') = \prod_{x=j+1}^{j'-1} (1 - P_{loc}(c, i, x)) \times P_{loc}(c, i, j') \times e^{-r\frac{v^2}{j'-x}}$$

So we have that for this car, the risk potential caused by its movement to (i, j) is

$$risk_{frontpotential}(c) = \sum_{j'=j+1}^{\infty} \prod_{x=j+1}^{j'-1} (1 - P_{loc}(c, i, x))$$
$$\times P_{loc}(c, i, j') \times e^{-r\frac{v^2}{j'-x}}$$

2. Assume the potential target cell of car c is (i, j) are about to calculate the potential risk between c and car c' that might appear at cell (i, j') and $j' \leq j$. This case is slightly different than the case above, because safety distance is not related to speed of c anymore, but that of the car c' that is right behind c.

We define that, for a cell (i, j), the probability that the car in that cell is c is

$$P_{car-at-cell}(c, i, j) = \frac{P_{car}(c, i, j)}{\sum_{all-car(c')} P_{car}(c', i, j)}$$

$$risk_{backpotential}(c) = \sum_{j'=j_0+1}^{j'-1} \prod_{x=j'+1}^{j-1} (1 - P_{loc}(c, i, x)) \times P_{loc}(c, i, j')$$
$$\times P_{caratcell}(c, i, j) \times e^{-r\frac{v_{c'}^2}{j-x}}$$
$$+ \prod_{x=j_0+1}^{j-1} P_{loc}(c, i, x) \times e^{-r\frac{v_0^2}{j-x}}$$

Where $v_{c'}$ is the speed of c' when it moves from its original cell to (i', j')

Finally, the total risk potential that car c will have by moving to cell (i, j) is

$$
\begin{aligned}
risk_{potential}(c) = risk_{frontpotential}(c) + risk_{backpotential}(c) = \\
\sum_{j'=j+1}^{\infty} \prod_{x=j+1}^{j'-1} (1 - P_{loc}(c, i, x)) \\
\times P_{loc}(c, i, j') \times e^{-r \frac{v^2}{j'-x}} \\
+ \sum_{j'=j_0+1}^{j'-1} \prod_{x=j'+1}^{j-1} (1 - P_{loc}(c, i, x)) \times P_{loc}(c, i, j') \\
\times P_{caratcell}(c, i, j) \times e^{-r \frac{v_{c'}^2}{j-x}} \\
+ \prod_{x=j_0+1}^{j-1} P_{loc}(c, i, x) \times e^{-r \frac{v_0^2}{j-x}}
\end{aligned}
$$

2.3.7 Summary

From all of above, we could calculate the value of $P_{car}(c, i, j)$, where

$$P_{car}(c, i, j) = z \times P_v(v) \times risk_{potential}(c) \times e^{|speed-change|}$$

where z is the coefficient corresponding to the lanes changes (0.45 for right, 0.35 for same, and 0.2 for left).

2.4 Flow and Safety

We initialize the locations of cars on the lattice grid map and speeds randomly, and then update the locations and speeds of cars in every iteration. We count the number of iterations needed so that all cars could leave the map. We also accumulate the risks caused by every movement that a car makes. In order to avoid variance of this method, we repeat the program for several times with same parameter. Then the average number of iterations could represents the flow and the risks could represents the safety issue. When average number of iterations is small then the flow is fast, and when the risk is smaller, then it is safer.

3 Result

Here are the fact that we find from our model:

1. As Figure 1 shows, when the density of car increases, the flow becomes slower and it takes more time to have cars left the road segment.

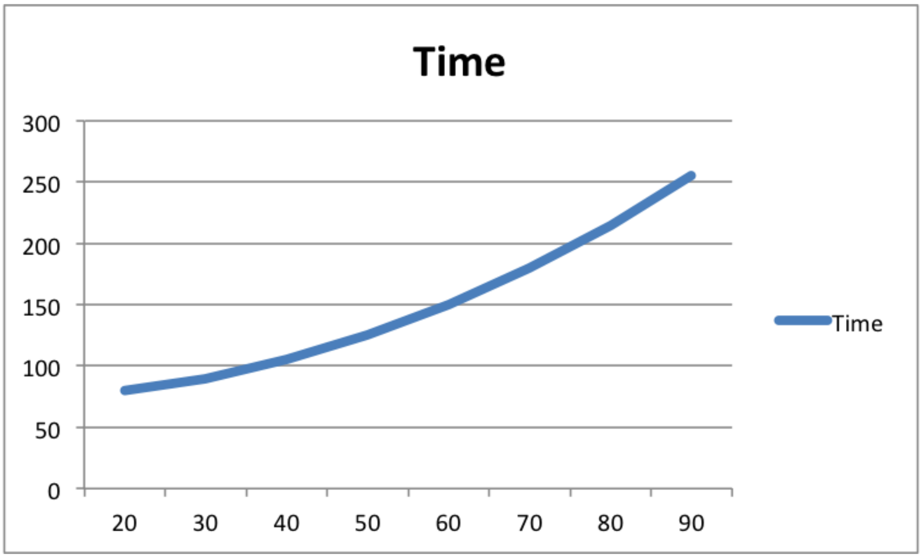

Figure 1: Car Density to Time

2. As Figure 2 shows, when the average speed increases, the risks increase relatively. So we could conclude that the average speed is proportion to the risk coefficient.

3. As Figure 3 shows, we can find from our graphs and output that the average speed of cars within our road segment has inverse relation with the time that the cars can pass by. And the actual time used under KRETPR is always slower than the expected time ($\frac{length}{speed}$)

Comparing the KRETPR with the rule that not limiting cars to move on the right-most line, we can find that the KRETPR is more likely to cause traffic for the following reasons:

1. Once a car changes line to the left, it will cause a viscosity occurs in the traffic and hence influence all the nearby cars.

2. All cars have speed depend on the leading car moving on the right-most line since the following cars have only two choices. The first one is to decrease speed to avoid collision and when the cars after the leading one speed down one-by-one, the traffic congestion will occur.

   However, if we do not apply the rule, the traffic flow can be separated into multiple and hence the right-most-line will not have heavy pressure.

For countries where driving automobiles on the left is the norm, our model will give the same results, because we only consider the probability that the driver will choose location (i, j).

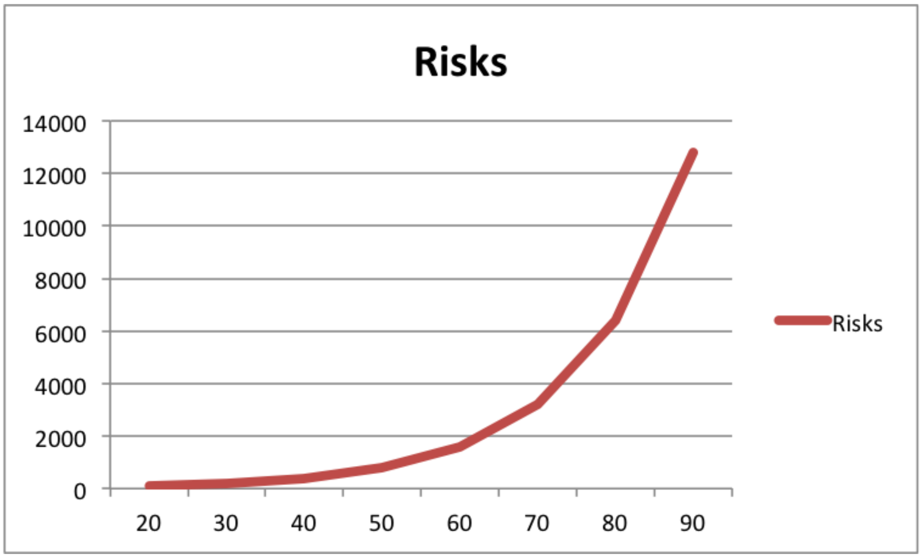

Figure 2: Car Speed to Risk

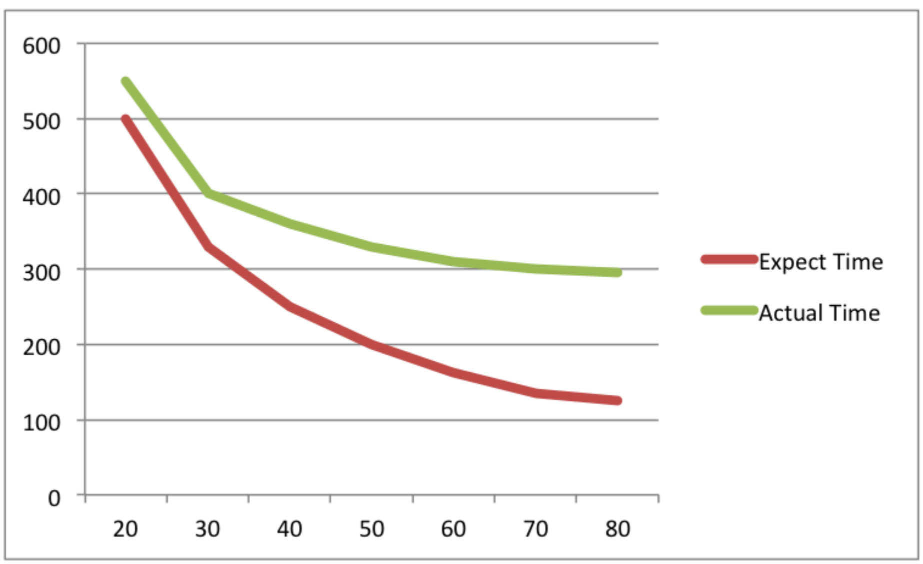

Figure 3: Car Speed to Time

When vehicle transportation on the same roadway was fully under the control of an intelligent system, I think there will be improvement on both flow and safety. Our model is based on driver's decision, which means that every driver only takes care about their time and safety. So the global flow and safety issue will be negatively affected by their own decision. However, when an intelligent system control the roadway, then it will optimize the global flow and safety regardless the issue of a specific person.

4 References

[1] Vehicle stopping distance and time.

[2] Chuan-Yang Yin Tao Zhou Bo Hu Kun Gao P.M. Hui Chin-Kun Hu Chuan-Ji Fu, Bing-Hong Wang. Analytical studies on a modifier nagel-schreckenberg model with the fukui-ishibashi acceleration rule. *Chaos, Solitons and Fractals*, 31:772–776, 2007.

[3] Houbing Song Yi-Chang Chiu, Liang Zhou. Development and calibration of the anisotropic mesoscopic simulation model for uninterruped flow facilities. *Transportation Research Part B*, 44:152–174, 2010.

[4] ZHANG Jun-ping ZENG Jing, HUANG Hai-yun. The research approach to the imitation of the traffic flows by hydrodynamics theories. *Journal of Guangzhou University (Natural Science Edition)*, 12(5):45–49, October 2013.