Math 465 Sample Problems

One notebook sized page of notes will be allowed on the test. Calculators in which no text, formulas or programs are stored should be brought to the test.

1. Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

   a) Compute the Sturm sequence of A.

   b) Use the Sturm sequence to determine how many eigenvalues of A are in $[0, 4]$.

2. Let

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 1 & -1 \\ 4 & -1 & 2 \end{bmatrix}$$

   a) Use Gerschgorin's theorem to locate the eigenvalues of A.

   b) Use $\|A\|_\infty$ to give an upper bound for the eigenvalues of A.

   c) Does A have any complex eigenvalues? Give reasons.

   d) Compute the Rayleigh quotient of A in the direction of the vector $[1, 1, 1]^T$.

3. Let

$$A = \begin{bmatrix} 0 & 4 & 1 \\ 4 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

   a) Compute the characteristic polynomial of A by Krylov's method.

   b) Compute the characteristic polynomial of A by Hyman's method.

4. Let

$$A = \begin{bmatrix} 2 & 12 & -5 \\ 12 & 1 & 2 \\ -5 & 2 & 1 \end{bmatrix}$$

   a) Either elimination or Householder transformations can be used to reduce A to Hessenberg form. Which would you use to preserve the symmetry of A?

   b) Only one Householder transformation Q is needed to reduce A to Hessenberg form. Q is of the form $Q = I - \frac{2}{|u|^2} uu^T$. Find u.

   c) Use the Q of part b) to reduce A to Hessenberg form.

5. Let $q(x) = x^4 + x^3 - 2x + 1$.

   a) Describe Bairstow's method for finding a real quadratic factor of a real polynomial.

   b) Beginning with $(u_0, v_0) = (1, 0)$, compute the first step of Bairstow's method applied to q.

   c) What is the companion matrix A of q?

   d) Apply Gerschgorin's circle theorem to the companion matrix to find an upper bound for all of the zeros of q.