Math 336 Sample $\mathfrak{Problems}$

One notebook sized page of notes will be allowed on the test. The test will cover up to §VI.2 in the text (excluding those sections for which there was no homework).

1. Let f be an analytic function on an open connected set W. Suppose $0 \in W$ and suppose $|f(\frac{1}{n})| < e^{-n}$ for all $n > 0$. Prove that $f(z) = 0$ for all $z \in W$.

2. Suppose $f(z)$ is an entire function and $|f(z)| < 1 + |z|^{1/2}$. Prove that f is constant.

3. Compute
$$\int_0^{2\pi} e^{e^{i\theta}} d\theta.$$

4. Suppose f is continuous on a connected open set W. Suppose also that f^2 and f^3 are analytic on W. Prove that f is analytic.

5. Let u and v be harmonic on an open connected set W. Suppose that $u(z)v(z) = 0$ on an open subset of W. Prove that either u or v is identically 0 on W.

6. Suppose $f(z) = u(z) + iv(z)$ is entire and $|u(z)| > |v(z)|$ for all z. Prove that f is constant.

7. Where does
$$\sum_{n=0}^{\infty} e^{-z^2\sqrt{n}}$$
converge? Where is it analytic?

8. Suppose f is analytic in $\{0 < |z| < r\}$ for some $r > 0$. Suppose also that $|f(z)| < |z|^{-1+\epsilon}$ in $\{0 < |z| < \delta\}$, where $\epsilon > 0$. Prove that f has a removable singularity at 0.

9. Let $D = \{z : |z| < 1\}$. Let f be analytic and non-constant on W, and suppose $\overline{D} \subset W$. Suppose $|f|$ is constant on ∂D. Prove that f has at least one zero in D.

10. Suppose $\mathrm{Re}(z_1) < 0, \mathrm{Re}(z_2) < 0$. Prove that
$$|e^{z_1} - e^{z_2}| < |z_1 - z_2|.$$

11. Suppose f is entire and $|f(z)| \leq |Ke^z|$ for some K. Prove that $f(z) = Ce^z$ for some C.

12. Let $\displaystyle\sum_{n=-\infty}^{\infty} a_n z^n$ of Laurent series of $\dfrac{1}{\sin(z)}$ in $|z| < \pi$. Prove that $a_n = 0$ if $n < -1$ and $a_n = 0$ if n is even. Compute a_{-1} and a_1.

13. There may be homework problems, example problems from the text, or requests for statements of theorems on the midterm.