Math 335 Sample Problems

One notebook sized page of notes will be allowed on the test. The test will cover from §7.1 to §8.2

1. Express the following integrals in terms of gamma functions.

   (a) $\int_0^\infty x^5 e^{-x^2}\, dx$

   (b) $\int_0^\infty x^{10} e^{-x^3}\, dx$

2. Let $f(x)$ be the odd 2π-periodic function that equals $\cos x$ on $(0, \pi)$. Find the Fourier series of f. What does it converge to? Draw a graph of f.

3. (a) Show that $c(x) = \sum_0^\infty \frac{x^{2n}}{(2n)!}$ converges uniformly on every finite interval $[a, b]$.

   (b) Prove that $c(x)$ is continuous everywhere.

   (c) Prove that $c(x)$ is differentiable everywhere.

4. (a) Let

   $$f_n(x) = \frac{2x}{\pi} \arctan(nx)$$

   (b) Prove that $\lim_{n\to\infty} f_n'(x)$ exists on $[-1, 1]$, but the convergence is not uniform.

5. Assume $\int_0^\infty |F(t)|dt < \infty$, $\int_0^\infty |F'(t)|dt < \infty$, and $\lim_{t\to\infty} e^{-st}F(t)$ for $s > 0$. Prove that

   (a) $\int_0^\infty e^{-st}F(t)dt$ and $\int_0^\infty e^{-st}F'(t)dt$ converge uniformly for $s \geq \delta > 0$.

   (b)

   $$\int_0^\infty e^{-st}F(t)dt = \frac{1}{s}F(0) + \frac{1}{s}\int_0^\infty F'(t)dt$$

6. You will need to know the definitions of the following terms and statements of the following theorems.

   1. Uniform convergence of a sequence or series
   2. Weierstrass M-test
   3. Continuity of a uniform limit of continuous functions
   4. Term-by-term integration
   5. Term-by-term differention
   6. Power series
   7. Radius of convergence of a power series
   8. Integration and differentiation of a power series
   9. Improper integrals dependent on a parameter
   10. Gamma function
   11. Fourier series
   12. Convergence of Fourier series

7. There may be homework problems or example problems from the text on the midterm.