8.C. First note that if f is non-constant, then it has at most finitely many zeros. For otherwise, the zeros have an accumulation point $b \in D$. By continuity $f(b) = 0$, contradicting the theorem that zeros of non-constant analytic functions are isolated. As we have seen before, if $|z| = 1$ and $|a| < 1$, then

$$\left| \frac{z-a}{1-\overline{a}z} \right| = 1.$$

If $\{a_j\}_1^n$ are the zeros of f where each zero is repeated in this list according to its multiplicity, let

$$B(z) = \prod_1^n \frac{z-a_j}{1-\overline{a_j}z}.$$

Then $g(z) = f(z)/B(z)$ is analytic in D because the singularities at a_j are removable. Moreover $|g(z)| \to 1$ as $|z| \to 1$, so that by the maximum principle $|g(z)| \leq 1$ on D. But $1/g(z)$ is also analtyic on D, because g is non-zero on D, and by the same reasoning $|1/g(z)| \leq 1$ on D. Thus $|g(z)| = 1$ on D. Because $|g|$ achieves its maximum value in D, g must be constant, by the maximum principle.