8.B There are several ways to solve this problem. Here is one: We first claim that $|\varphi(z)| \to 1$ as $|z| \to 1$. If so, then by the solution to problem 8C, φ is given by a product. But because φ is one-to-one, it can have only one zero, and thus it must have the desired form (1). We need only prove the claim. If the claim is false, then there is a sequence $a_j \in D$ with $|a_j| \to 1$ and with $\varphi(a_j) \to b \in D$. Because φ is onto, there exists $c \in D$ such that $\varphi(c) = b$. Because analytic functions are open, (proved in class), the image of a disk $B(c)$ centered at c contains a disk $B'(b)$ centered at b. We may suppose that $\overline{B(c)}$ is contained in D, so that $a_j \notin B(c)$ for $j \geq j_0$, if j_0 is sufficiently large. But for sufficiently large j_0, $\varphi(a_{j_0}) \in B'(b)$ and there is then a $d \in B(c)$ so that $\varphi(d) = \varphi(a_{j_0})$. In particular $d \neq a_{j_0}$. This contradicts the assumption that φ is one-to-one, and completes the proof of problem 8B.

Note that the claim really does not depend on analyticity nor on the region being the disk. If f is a one-to-one open continuous map of a connected open set U onto $f(U)$ then $f(z) \to \partial f(U)$ as $z \to \partial U$. A map with this property is called "proper". So one-to-one analytic maps are proper.