Math 427 Winter 2023

Homework 8

Due Wednesday March 8

Section 3.4 12,15,16

Section 3.5: 3, 5

and

In the following problems let $D = \{z : |z| < 1\}$ denote the open unit disk.

A.1 If c and a are constants with $|c| = 1$ and $|a| < 1$, prove that

$$\varphi(z) = c\left(\frac{z-a}{1-\overline{a}z}\right) \tag{1}$$

is a one-to-one analytic map of D onto D. Hint: explicitly find the inverse function.

A.2 Show that φ^{-1} is of the same form as φ, but with different constants.

A.3 Show that $|\varphi(z)| = 1$ when $|z| = 1$.

B. Suppose φ is a one-to-one analytic map of the open unit disk D onto D. Prove that there are constants c and a with $|c| = 1$ and $|a| < 1$ so that

$$\varphi(z) = c\left(\frac{z-a}{1-\overline{a}z}\right). \tag{1}$$

Hint: there are two typical ways to prove $f = g$: show $f - g = 0$ or show $f/g = 1$. Careful: φ is not defined on $|z| = 1$.

C. Suppose f is analytic in D and suppose $|f(z)| \to 1$ as $|z| \to 1$. Prove f is rational.

D. Suppose f is analytic in D and suppose $|f(z)| < 1$ in D. Let $a = f(0)$. Show that $f(z) \neq 0$ if $|z| < |a|$. Hint: Use f to build another function g with $g(0) = 0$ and $|g| < 1$ on D.