

Math 427 Winter 2023

### Homework 6

Due Mon. Feb. 13

Section 3.1: 6, 7, 15

Section 3.2: 6, 8, 9 10, 14

Plus:

- A. If  $f$  has a power series expansion at  $z_0$  with radius of convergence  $R$ , and if  $r < R$  then there is a constant  $C$  so that  $|f(z) - f(z_0)| \leq C|z - z_0|$  provided  $|z - z_0| \leq r$ .
- B. Use the same idea as in A to show that if  $f(z) = \sum a_n(z - z_0)^n$  then

$$\left| f(z) - \sum_{n=0}^k a_n(z - z_0)^n \right| \leq D_k |z - z_0|^{k+1},$$

where  $D_k$  is a constant and  $|z - z_0| \leq r < R$ .

- C. Use the proof of the root test to give an explicit estimate of  $D_k$  (for large  $k$ ) and therefore an estimate of the rate of convergence of the series for  $f$  if  $|z - z_0| < r < R$ .