Math 427 Winter 2023

Homework 1

Due Weds January 11 at the beginning of class

In the problems below z and w are complex numbers.

1. Prove $\overline{1/z} = 1/\overline{z}$ and $|1/z| = 1/|z|$ and if $z \neq 0$ then $|z/\overline{z}| = 1$.

2. Prove the parallelogram equality:

$$|z + w|^2 + |z - w|^2 = 2(|z|^2 + |w|^2),$$

for complex numbers z, w using complex notation. In geometric terms, the equality says that for the parallelogram with vertices 0, z, w, $z+w$, the sum of the squares of the lengths of the diagonals is equal to the sum of the squares of the lengths of the edges. Your proof should be much easier that the traditional proof using high school geometry.

3. Prove $|z - w| \geq ||z| - |w||$ for all complex numbers z, w.

4. Prove that if $\{z_n\}$ does not have limit 0, then $\sum z_n$ diverges.

5. Where does

$$\sum_{n=1}^{\infty} \frac{z^n}{2^n - 1}$$

converge and where does it diverge? Prove your answers.

6. Where does

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + z^2}$$

converge? Prove your answer. (Be careful: there are values of z where the series is not defined).