Midterm 2 – Math 126

Full Name	NetID

- This midterm is 50 minutes long. It contains 5 independent problems over 12 pages. Problems 1 and 5 have two parts.
- You may use a A4, double-sided handwritten cheat sheet, as well as a Ti-30x IIS Calculator, but nothing else.
- You won't get any credit for answers without work or mathematical justification. A picture is not a mathematical justification.
- You also won't get any credit for a correct answer but a thoroughly wrong reasoning.
- Make sure to write neatly: we cannot grade what we can't read!
- For full credit, your results must be fully simplified. For instance, an equation of the form $(x-1)^2 + y = x^2$ needs to be reduced to $-2x + y + 1 = 0$.
- You should write your solutions on the pages directly following the problem statement. If you end up using some scrap pages at the end, please indicate it clearly.

Problem 1. (20 points) *The two questions are independent.* Let f be the function given by:

$$f(x, y) = 1 + \sqrt{x^2 - 2xy}.$$

(a) (10 points) Find the domain of f and plot it in the xy-plane.

(b) (10 points) Find the equation of the tangent plane to the graph of f at the point $(1, 0)$.

Problem 2. (20 points) f be the function given by:

$$f(x, y) = \sqrt{x^2 + 3xy}.$$

Using the linear approximation of f at $(10, 10)$, find an approximate value for $f(11, 12)$.
You will not get any credit if you compute $f(11, 12)$ *using your calculator instead of the linear approximation.*

Problem 3. (20 points) Let f be the function given by:

$$f(x,y) = x^3 + y^3 - 3xy$$

Find all critical points of f and identify whether they correspond to local minima, local maxima or saddle points.

Problem 4. (20 points) Let f be the function given by:

$$f(x, y) = x^2 + 2y^2 + 8y.$$

Find the global minimum and the global maximum of f in the set

$$S = \{(x, y) \mid \; x^2 + y^2 \leq 16\}.$$

Problem 5. (20 points) *The two questions are independent.* Let $\vec{r}(t)$ be the curve of equation

$$\vec{r}(t) = \left\langle 4t, t^2, t^3 \right\rangle .$$

(a) (10 points) Find a point on this curve whose tangent line is parallel to the line of equation

$$\begin{cases} x = 4t \\ y = -2t \\ z = 3t \end{cases} .$$

(b) (10 points) Find the curvature of $\vec{r}(t)$ at the point $B(0, 0, 0)$.