1. **[13 points]** Consider the function $f(x, y) = 6xy + x^3 - y^2$.

   Find all critical points for f, then classify each one as a local maximum, local minimum, or saddle point.

$$f_x(x,y) = 6y + 3x^2 = 0 \to 6(3x) + 3x^2 = 0 \to 3x(6+x) = 0$$

$$f_y(x,y) = 6x - 2y = 0 \to y = 3x$$

$x = 0$ or $x = -6$

$y = 0$ $\quad$ $y = -18$

$(0,0)$ & $(-6,-18)$

$f_{xx}(x,y) = 6x$

$f_{yy}(x,y) = -2$

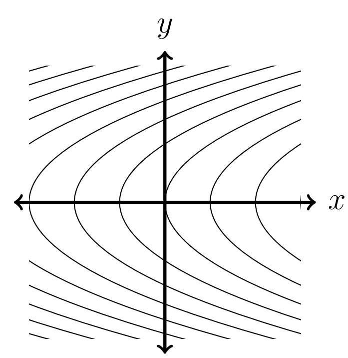
$f_{xy}(x,y) = 6$

$D(0,0) = 0 - 6^2 < 0$, so $\boxed{\text{saddle point @ } (0,0)}$

$D(-6,-18) = (-36)(-2) - 6^2 > 0$, and $-36 < 0$, so $\boxed{\text{local max @ } (-6,-18)}$

2. **[4 points]** Give an example of a function $f(x, y)$ whose level curves are all parabolas that look like this:

   Just write a formula for f. You do not need to show work.
   There are many possible answers.



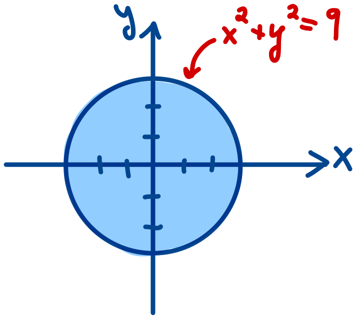
Level curves look like $x = y^2 + k \to k = x - y^2$

So $\boxed{f(x,y) = x - y^2}$ works

(other answers are possible)

3. **[16 points]** Let $\mathcal{D}$ be the closed disc of radius 3 centered at the origin.

Find the absolute minimum and maximum values of $f(x,y) = x^2 + 4x + 2y^2$ on $\mathcal{D}$.



Crit pts: $f_x(x,y) = 2x+4 = 0 \rightarrow x=-2$

$f_y(x,y) = 4y = 0 \rightarrow y = 0$

Only crit. pt. is $(-2,0)$

Boundary: $x^2+y^2=9 \rightarrow y^2 = 9-x^2$

So $f(x,y) = x^2+4x+2(9-x^2) = -x^2+4x+18$

on domain $\underbrace{-3 \le x \le 3}$. $\rightarrow$ Check endpoints $(-3,0)$ & $(3,0)$

$f'(x) = -2x+4 = 0$

$\downarrow$

$x=2$, so $2^2+y^2=9$

$\downarrow$

$y = \pm\sqrt{5}$

Points to check

$(-2,0)$

$(-3,0)$

$(3,0)$

$(2,\sqrt{5})$

$(2,-\sqrt{5})$

$f(-2,0) = \boxed{-4}$ min

$f(-3,0) = -3$

$f(3,0) = 21$

$f(2,\pm\sqrt{5}) = \boxed{22}$ max

4. (a) **[8 points]** Find the equation of the plane tangent to $z = xy^2 - \sqrt{x} - 3\sin(y-2)$ at the point $(4, 2, 14)$.

$$\frac{\partial z}{\partial x} = y^2 - \frac{1}{2\sqrt{x}}$$

$$\frac{\partial z}{\partial y} = 2xy - 3\cos(y-2)$$

$x=4$
$y=2$

$$\frac{\partial z}{\partial x} = \frac{15}{4}$$

$$\frac{\partial z}{\partial y} = 13$$

$$\boxed{z = \frac{15}{4}(x-4) + 13(y-2) + 14}$$

(b) **[5 points]** Use your answer to part (a) to find an approximate value of y that satisfies the following equation:

$$\underbrace{14.22}_{z} = \underbrace{3.92}_{x}y^2 - \sqrt{\underbrace{3.92}_{x}} - 3\sin(y-2)$$

$$14.22 = \frac{15}{4}(3.92-4) + 13(y-2) + 14$$

$$0.22 = -0.3 + 13(y-2)$$

$$0.52 = 13(y-2)$$

$$0.04 = y-2$$

$$\boxed{y = 2.04}$$

5. **[7 points per part]** For each of the following prompts, write the indicated iterated integral.

<u>**Do not try to evaluate these integrals!**</u> Just set them up as instructed.

(a) Write an iterated integral for the volume below the surface $z = e^x + y^3$, above the surface $z = 1 + \sin(y)$, and over the rectangle $[3, 5] \times [2, 4]$ in the xy-plane.

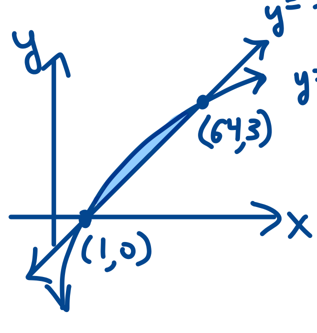
Integrand = top - bottom = $(e^x + y^3) - (1 + \sin y)$

$$\boxed{\int_3^5 \int_2^4 \left(e^x + y^3 - 1 - \sin(y)\right) dy\, dx}$$

(b) Rewrite $\displaystyle\int_1^{64} \int_{\frac{x-1}{21}}^{\log_4 x} x^2 \sin(y^3)\, dy\, dx$ by reversing the order of integration.

$y = \dfrac{x-1}{21} \longleftrightarrow x = 21y + 1$

$y = \log_4 x \longleftrightarrow x = 4^y$



$$\boxed{\int_0^3 \int_{4^y}^{21y+1} x^2 \sin(y^3)\, dx\, dy}$$