Midterm 1 – Math 126

- This midterm is 50 minutes long. It contains 5 independent problems over 24 pages; (most are blank!
- You may use a A4, double-sided handwritten cheat sheet, as well as a Ti-30x IIS Calculator, but nothing else.
- You won't get any credit for answers without work or mathematical justification.
- Make sure to write neatly: we cannot grade what we can't read!
- For full credit, your results must be fully simplified. For instance, an equation of the form $(x-1)^2 + y = x^2$ needs to be reduced to $-2x + y + 1 = 0$.
- You should write your solutions on the pages directly following the problem statement. If you end up using some scrap pages at the end, please indicate it clearly.

Problem	Score
1	
2	
3	
4	
5	
Total	

Problem 1. The two parts are *not* independent.
(a) (10 points) Show that the points $A(0,0,1)$ and $B(1,2,3)$ belong to the curve of equation

$$\vec{r}(t) = \left\langle t^3, 2t^3, 2t^3 + 1 \right\rangle .$$

(b) (10 points) Find the length of the arc of the curve $\vec{r}(t)$ between $A(0,0,1)$ and $B(1,2,3)$.

Problem 2. (20 points) Find the intersection of the two lines of equation:

$$\begin{cases} x = 2 + t \\ y = 3 - t \\ z = 1 - 2t \end{cases} \quad \text{and} \quad \begin{cases} x = 1 + t \\ y = 1 + 2t \\ z = 4 - 3t \end{cases}.$$

Hint: be careful about setting the right equation!

Problem 3. The two parts are independent.

(a) (10 points) Find a vector-valued function whose graph is the intersection of the surfaces of equations:

$$x^2 + (y+2)^2 = 9 \quad \text{and} \quad x - z = 1.$$

(b) (10 points) Find the equation of the line that, at $t = 0$, is tangent to the graph of the vector-valued function

$$\vec{r}(t) = \left\langle -t, \cos(2t), 3t^2 + 3t \right\rangle .$$

Problem 4. (20 points) Find a parametric equation for the intersection of the two planes given by the equations $2x + y - z = 1$ and $x + 3y - z = 0$.

Problem 5. (20 points) Let $A(0,0,0)$ and $B(0,2,0)$ and $\mathcal{S}$ be the sphere of radius 1, centered at A. Find an equation for the set of all points C on the sphere $\mathcal{S}$ that maximize the area of the triangle ABC.