Math 126 D Autumn 2024 Midterm I

October 22, 2024

Name＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿

Student Number＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿＿

Instructions

- These exams will be scanned. **Please write your name and student number clearly.**

- There are 4 questions. The exam is out of 50 points.

- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. No printed material allowed. **Hand in your notes with your exam.**

- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ($\frac{2\ln 3}{\pi}$ and 1/3 are exact, 0.699 and 0.333 are approximations for those numbers.)

- Show your work. If we cannot read or follow your work, we cannot grade it. **You may not get full credit for a right answer if your answer is not justified by your work.**

1. The parallelogram on the right has three of its corners given by $A(2, -1, 3)$, $B(5, -2, 2)$, and $D(3, 2, 1)$.
   The picture is not to scale.
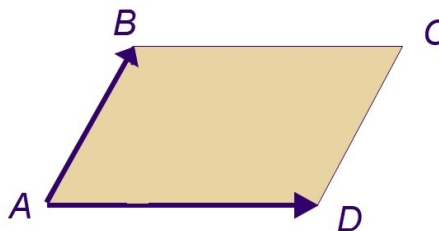
(a) (4 points) Draw the two diagonals of the parallelogram. Given that they bisect each other (i. e. they meet each other exactly at their middle points) find the point of their intersection.

(b) (4 points) Find the acute angle between the two diagonals of the parallelogram.

(c) (5 points) Find the area of the parallelogram.

2. Two planes are given by

$$2x - 3y = 9 \quad \text{and} \quad x + 2y - z = -4$$

(a) (7 points) Find a vector equation for the line of intersection of the two planes.

Before moving on, check that your line is on both planes.

(b) (5 points) Find the equation of a third plane which contains this line and the point $(3, 1, -4)$. Given your answer in the standard form $Ax + By + Cz = D$.

3. (3 points each) The following parts are not related.

    (a) An ostrich egg is is shown on the right. The circumference at the widest circle is 34 centimeters. The height is 20 centimeters. Write down the equation of an ellipsoid with center at the origin which would model the surface of this egg with z-axis vertical.

EGGSHELL

Height
20 cm

Circumference
34 cm
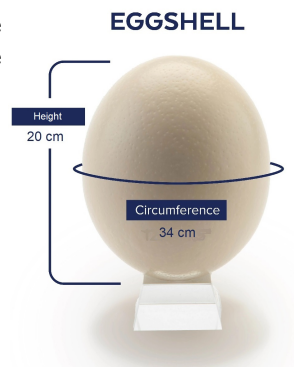
    (b) Write down the equation of a parabolic cylinder which is symmetric with respect to the yz-plane (yz-plane acts like a mirror).

    (c) Write down parametric equations of a helix/spiral that wraps around the elliptic cylinder

$$\frac{x^2}{4} + \frac{z^2}{9} = 1.$$

    (d) The sphere $(x-5)^2 + (y+3)^2 + (z-6)^2 = 4$ is tangent to the sphere $x^2 + y^2 + z^2 = r^2$. What are the two possible values of r?
There is help/hint on last page.

4. Given
$$\mathbf{r}(t) = \langle 2t + 1, t - 5, t^2 - t + 2 \rangle$$

(a) (7 points) Find a vector equation of the tangent line to this curve at the point $(7, -2, 8)$.

(b) (3 points) Compute the curvature $\kappa(t)$.

(c) (3 points) Compute the maximum curvature $\kappa(t)$ and find the point where it occurs.

This page is blank. If you continued a question here, make a note on the question page so we can have a look.

Sphere hint: Think about this question: The circle $(x - 3)^2 + (y - 4)^2 = 4$ is tangent to the circle $x^2 + y^2 = a^2$ on the xy-plane. What are the two possible values of a? This will not be graded.