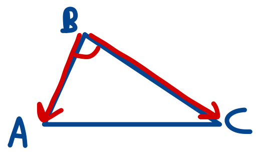
1. **[4 points per part]** For this problem, consider the points

$$A = (1, 4, -1) \qquad B = (-1, 3, 1) \qquad C = (5, 1, 4).$$

(a) Compute the angle $\angle ABC$.



$\vec{BA} = \langle 2, 1, -2 \rangle \quad |\vec{BA}| = \sqrt{4+1+4} = 3$

$\vec{BC} = \langle 6, -2, 3 \rangle \quad |\vec{BC}| = \sqrt{36+4+9} = 7$

$\vec{BA} \cdot \vec{BC} = 12 - 2 - 6 = 4$

$\vec{BA} \cdot \vec{BC} = |\vec{BA}||\vec{BC}| \cos\theta$

$4 = 3 \cdot 7 \cos\theta$

$\theta = \boxed{\cos^{-1}\left(\frac{4}{21}\right)} \approx 1.379 \text{ radians}$

$\approx 79.02°$

(b) Write parametric equations for the line through A which is parallel to the line $\overline{BC}$.

Point: $A = (1, 4, -1)$

Direction: $\vec{BC} = \langle 6, -2, 3 \rangle$

$$\boxed{\begin{aligned} x &= 1 + 6t \\ y &= 4 - 2t \\ z &= -1 + 3t \end{aligned}}$$

(c) Find the equation of the plane through A, B, and C.

$\vec{BA} \times \vec{BC} = \langle 2, 1, -2 \rangle \times \langle 6, -2, 3 \rangle = \langle -1, -18, -10 \rangle$

Point: $(1, 4, 1)$ $\quad -1(x-1) - 18(y-4) - 10(z+1) = 0$

or

$$\boxed{x + 18y + 10z = 63}$$

2. **[8 points]** Find parametric equations for the line through $(0, 1, 2)$ which is perpendicular to the line $\mathbf{r}(t) = \langle 1 + 4t, 3t, 2 - t\rangle$ and parallel to the plane $x - 6y + 2z = 4$.

Orthogonal to $\langle 4, 3, -1\rangle$ Also orthog. to $\langle 1, -6, 2\rangle$

Direction vector: $\langle 4,3,-1\rangle \times \langle 1,-6,2\rangle = \langle 0,-9,-27\rangle$ or, scaled down by $\frac{-1}{9}$, $\langle 0,1,3\rangle$

Through point $(0,1,2)$:

$$\boxed{\begin{aligned} x &= 0 \\ y &= 1 + t \\ z &= 2 + 3t \end{aligned}}$$

3. **[3 points per point]** For each of the following prompts, give an example of a vector function that meets the condition. (There are many possible answers! Just write one of them.)

You do not need to show any work.

(a) A vector function whose space curve intersects the plane $z = 2$ exactly twice.

Lots of ways. One example: $\boxed{\vec{r}(t) = \langle t, t, t^2\rangle}$

note: x- and y-coords at both intersections shouldn't be equal

(b) A vector function whose normal component of acceleration is always 0.

Space curve should be a line, e.g. $\boxed{\vec{r}(t) = \langle 0, t^3, 0\rangle}$

(or even just parametric eq's for a line)

(c) A vector function whose space curve has a constant curvature of 7.

Easiest way: circle of radius $\frac{1}{7}$, like $\boxed{\vec{r}(t) = \langle \frac{1}{7}\cos t, \frac{1}{7}\sin t, 0\rangle}$

4. **[5 points per part]** Let $\mathcal{S}$ be the surface $x^2 - 2x - y^2 - 4y + z^2 = -2$.

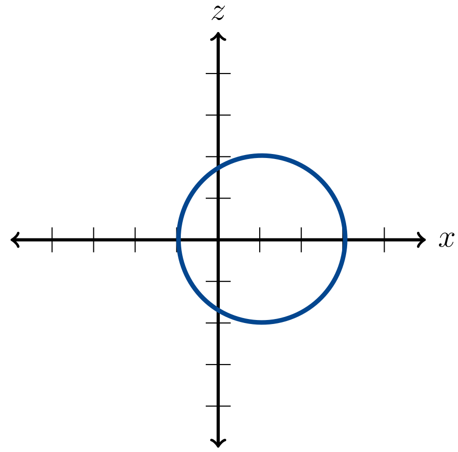
(a) Draw the trace of $\mathcal{S}$ in the plane $y = 1$.

$$x^2 - 2x - 1 - 4 + z^2 = -2$$

$$x^2 - 2x + 1 + z^2 = 3 + 1$$

$$(x-1)^2 + z^2 = 4$$

Circle of radius 2
centered @ $x=1$, $z=0$



(b) Write the name of $\mathcal{S}$. (Show your work!)

Note: this problem does not rely on part (a).

$$x^2 - 2x + 1 - y^2 - 4y - 4 + z^2 = -2 + 1 - 4$$

$$(x-1)^2 - (y+2)^2 + z^2 = -5$$

$$\underbrace{-\frac{(x-1)^2}{5} + \frac{(y+2)^2}{5} - \frac{z^2}{5} = 1}$$

$\boxed{\text{hyperboloid of 2 sheets}}$

5. **[6 points]** Write a vector function whose space curve is the intersection of the surfaces

$$x^2 - y + 3z = 7 \qquad \text{and} \qquad \underbrace{x^2 + z^2 = 4}_{\text{parametrize as a circle}}.$$

$$y = x^2 + 3z - 7$$

$x = 2\cos t$

$z = 2\sin t$

$y = 4\cos^2 t + 6\sin t - 7$

$$\boxed{\vec{r}(t) = \left\langle 2\cos t,\ 4\cos^2 t + 6\sin t - 7,\ 2\sin t \right\rangle}$$

6. **[5 points per part]** Use the vector function $\mathbf{r}(t) = \langle t^2 - t, 2t^4, 1 - 4t\rangle$ to answer parts (a)–(c).

(a) Find parametric equations for the line tangent to the space curve of $\mathbf{r}(t)$ at $(6, 32, 9)$.

$\vec{r}\,'(t) = \langle 2t-1, 8t^3, -4\rangle$

$\vec{r}\,'(-2) = \underbrace{\langle -5, -64, -4\rangle}_{\text{direction vector}}$

$1 - 4t = 9$

$t = -2$

$$\boxed{\begin{aligned} x &= 6 - 5t \\ y &= 32 - 64t \\ z &= 9 - 4t \end{aligned}}$$

(b) Compute $\mathbf{T}(1)$, the unit tangent vector to the curve at $t = 1$.

$\vec{r}\,'(1) = \langle 1, 8, -4\rangle$

$|\vec{r}\,'(1)| = \sqrt{1 + 64 + 16} = 9$

$\vec{T}(1) = \dfrac{\vec{r}\,'(1)}{|\vec{r}\,'(1)|} = \boxed{\left\langle \frac{1}{9}, \frac{8}{9}, \frac{-4}{9} \right\rangle}$

(c) A 3-kilogram object's position vector after t seconds is $\mathbf{r}(t)$ meters. Find the force applied to it at time $t = 1$.

$\vec{a}(t) = \vec{r}\,''(t) = \langle 2, 24t^2, 0\rangle$

$\vec{a}(1) = \vec{r}\,''(1) = \langle 2, 24, 0\rangle$

$\vec{F}(1) = m\vec{a}(1) = 3\langle 2, 24, 0\rangle = \boxed{\langle 6, 72, 0\rangle \text{ Newtons}}$