Math 126 E - Autumn 2023
Midterm Exam Number One
October 26, 2023

Name: _____   Student ID no. : _____

Signature: _____

1	12
2	8
3	9
4	10
5	6
6	15
Total	60

This grid is purely decorative.
The exam is graded online.

- This exam consists of **SIX** problems on **FOUR** double-sided pages. The fourth page is left blank for scratch work.

- Show all work for full credit.

- You may use a TI-30X IIS calculator during this exam. Other calculators and electronic devices are not permitted.

- You do not need to simplify your answers, except where noted.

- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.

- ☐ Draw a box ☐ around your final answer to each problem.

- **Do not write within 1 centimeter of the edge!** Your exam will be scanned for grading.

- If you run out of room, write on one of the scratch work pages **and indicate that you have done so**. If you still need more room, raise your hand and ask for an extra page.

- You may use one hand-written double-sided 8.5" by 11" page of notes.

- You have 50 minutes to complete the exam.

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle "see first page" below a problem.

1. **[4 points per part]** For this problem, consider the points

$$A = (1, 4, -1) \qquad B = (-1, 3, 1) \qquad C = (5, 1, 4).$$

(a) Compute the angle $\angle ABC$.

(b) Write parametric equations for the line through A which is parallel to the line $\overline{BC}$.

(c) Find the equation of the plane through A, B, and C.

2. **[8 points]** Find parametric equations for the line through $(0, 1, 2)$ which is perpendicular to the line $\mathbf{r}(t) = \langle 1 + 4t, 3t, 2 - t \rangle$ and parallel to the plane $x - 6y + 2z = 4$.

3. **[3 points per point]** For each of the following prompts, give an example of a vector function that meets the condition. (There are many possible answers! Just write one of them.)

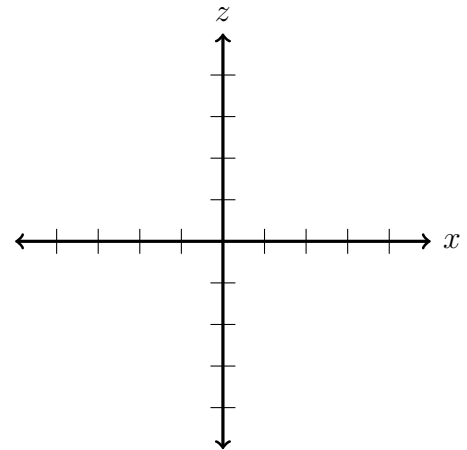   *You do not need to show any work.*

   (a) A vector function whose space curve intersects the plane $z = 2$ exactly twice.

   (b) A vector function whose normal component of acceleration is always 0.

   (c) A vector function whose space curve has a constant curvature of 7.

4. **[5 points per part]** Let $\mathcal{S}$ be the surface $x^2 - 2x - y^2 - 4y + z^2 = -2$.

   (a) Draw the trace of $\mathcal{S}$ in the plane $y = 1$.

   (b) Write the name of $\mathcal{S}$. (Show your work!)

   Note: this problem does not rely on part (a).

5. **[6 points]** Write a vector function whose space curve is the intersection of the surfaces

$$x^2 - y + 3z = 7 \qquad \text{and} \qquad x^2 + z^2 = 4.$$

6. **[5 points per part]** Use the vector function $\mathbf{r}(t) = \langle t^2 - t, 2t^4, 1 - 4t \rangle$ to answer parts (a)–(c).

(a) Find parametric equations for the line tangent to the space curve of $\mathbf{r}(t)$ at $(6, 32, 9)$.

(b) Compute $\mathbf{T}(1)$, the unit tangent vector to the curve at $t = 1$.

(c) A 3-kilogram object's position vector after t seconds is $\mathbf{r}(t)$ meters. Find the force applied to it at time $t = 1$.

You may use this page for scratch-work.

All work on this page will be ignored unless you write & circle "see back page" below a problem.

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