Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and stow away all cell phones, watches, pagers, music players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator. No other models are allowed.
- In order to receive credit, you must **show your work**. If you do not indicate the way in which you solved a problem, or if the work shown is incorrect or incomplete, you may get little or no credit for it, even if your answer is correct.
- You may use directly the integral formulas in the table below. You must **show your work in evaluating any other integrals**, even if they are on your sheet of notes.

Table of Integration Formulas Constants of integration have been omitted.

1. $\int x^n\,dx = \frac{x^{n+1}}{n+1} \quad (n \neq -1)$	**2.** $\int \frac{1}{x}\,dx = \ln\lvert x\rvert$
3. $\int e^x\,dx = e^x$	**4.** $\int b^x\,dx = \frac{b^x}{\ln b}$
5. $\int \sin x\,dx = -\cos x$	**6.** $\int \cos x\,dx = \sin x$
7. $\int \sec^2 x\,dx = \tan x$	**8.** $\int \csc^2 x\,dx = -\cot x$
9. $\int \sec x \tan x\,dx = \sec x$	**10.** $\int \csc x \cot x\,dx = -\csc x$
11. $\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert$	**12.** $\int \csc x\,dx = \ln\lvert \csc x - \cot x\rvert$
13. $\int \tan x\,dx = \ln\lvert \sec x\rvert$	**14.** $\int \cot x\,dx = \ln\lvert \sin x\rvert$
17. $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$	**18.** $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right), \quad a > 0$

- Place $\boxed{\text{a box around your answer}}$ to each question. Unless otherwise instructed, simplify your answers, but leave them in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$).
- All pages are double-sided except for this cover page and the last page. You may use the blank pages for extra room but, if you want us to grade these spare pages, clearly **indicate in the problem area** that your work is on the back of the cover page or on the blank page(s) at the end of the exam.
- This exam has 10 problems on 10 pages. When the exam starts, check that your exam is complete. Good luck!

1. (10 points) Evaluate the following integrals. Show your work. Simplify and box your answers.

   (a) $\displaystyle\int \tan^5(x)\sec^7(x)\,dx$

   (b) $\displaystyle\int \frac{2}{(x^2+16)^{3/2}}\,dx$

2. (10 points) Evaluate the following integrals. Show your work. Simplify and box your answers.

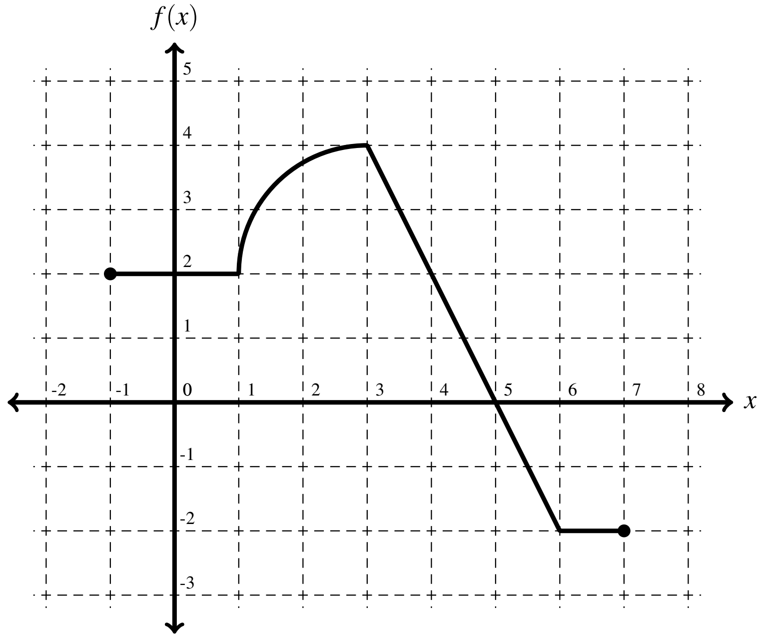
(a) $\int_0^1 x \tan^{-1}(x)\, dx$

(b) Evaluate the improper integral: $\int_0^\infty \frac{5}{3x^2+6x+6}\, dx$

3. (10 points) A tomato is dropped from the top of a tall building. You do not know when it was dropped, but you see the tomato go by your window, which is 320 feet above the ground. Two seconds after it passes your window, the tomato hits the ground. (The acceleration due to gravity is 32 ft/sec^2.)

   How tall is the building? (*Use Calculus methods, and show your work.*)

4. (10 points) The following is the graph of a function $y = f(x)$ over the domain $[-1, 7]$. It consists of line segments and a quarter circle. Use it to answer the questions in parts (a)-(d) below.



(a) Let $g(x) = \int_{-1}^{\sqrt{x}} f(t)\,dt$. Find $g'(9)$.

(b) Compute $\lim_{n\to\infty} \sum_{i=1}^{n} \left(f\left(\frac{3i}{n}\right) \frac{6}{n} \right)$.

(c) Evaluate $\int_{3}^{4} x f(x^2 - 10)\,dx$.

(d) Find the average value of $f(x)$ on the interval $[1, 3]$.

5. (10 points) The region in the xy-plane that is above the x-axis, below the curve $y = \sqrt{x}$, and between the lines $x = 1$ and $x = 4$, is rotated around the horizontal line $y = -3$ to form a solid of revolution.

   (a) Using washers, set up a definite integral for the volume of this solid, and evaluate this integral to find the volume.

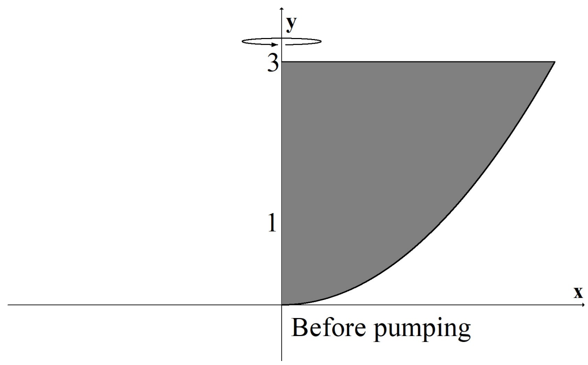
   (b) Using shells, set up definite integral(s) for the volume of this solid, but DO NOT EVALUATE THESE INTEGRAL(S).

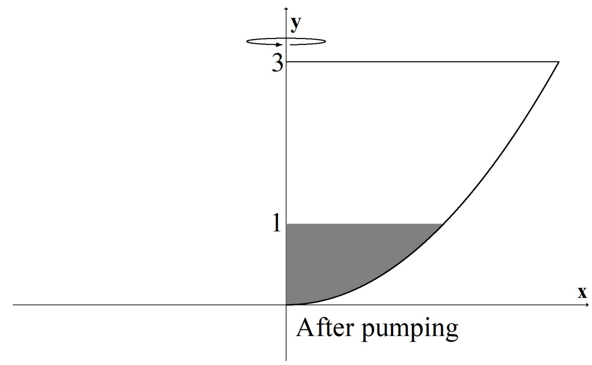
6. (10 points) This question refers to the arc length of the curve $y = e^{x^2}$ on the interval $[1,7]$.

   (a) Set up an integral equal to the arc length of the curve $y = e^{x^2}$ over the interval $[1,7]$. Do not evaluate the integral.

   (b) Approximate the arc length of $y = e^{x^2}$ on the interval $[1,7]$ via Simpson's Rule with $n = 6$ subintervals. (Please leave your answer in simplified exact form, rather than writing a decimal.)

7. (10 points) The region bounded by $y = x^2$, $x = 0$, and $y = 3$ is rotated about the y-axis to form a container. Distances are measured in feet. Initially, the container is full of a fluid that has a density of 50 lbs/ft^3. How much work is done (in ft-lb) to pump the top 2 feet of liquid to the top of the container?





8. (10 points) Find the x-coordinate ($\bar{x}$) of the centroid of the region enclosed by:

$$y = \frac{1}{9 - x^2}, \quad x = 0, \quad x = 2, \quad \text{and the } x\text{-axis.}$$

9. (10 points) Find the explicit solution $y = f(x)$ of the initial value problem:

$$\frac{dy}{dx} = x\sqrt{16 - y^2}, \qquad y(0) = 2.$$

10. (10 points) A type of bacteria grows at a rate proportional to its population, so it is governed by the differential equation $y' = \alpha y$ where $y = y(t)$ is the number of bacteria after t days and α is a constant.

 (a) If there are initially 12 bacteria and after 2 days there are 130 bacteria, what is α?

 (b) The scientists now (after the first 2 days) start harvesting 100 bacteria every day, so from now on the equation $y' = \alpha y - 100$ models the situation. If α is as in part (a), how many days (after the initial time) until there are 1,000 bacteria?