Worksheet for Week 3: Graphs of $f(x)$ and $f'(x)$

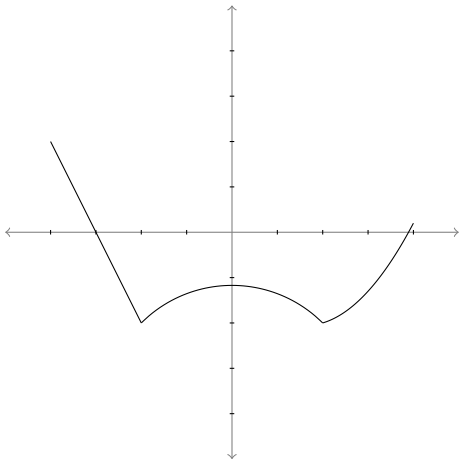
In this worksheet you'll practice getting information about a derivative from the graph of a function, and vice versa. At the end, you'll match some graphs of functions to graphs of their derivatives.

If $f(x)$ is a function, then remember that we define

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

If this limit exists, then $f'(x)$ is the slope of the tangent line to the graph of f at the point $(x, f(x))$.

Consider the graph of $f(x)$ below:



1. Use the graph to answer the following questions.
   (a) Are there any values x for which the derivative $f'(x)$ does *not* exist?

   Solution: The derivative doesn't exist where there are "corners" in the graph; in this case, at $x = -2$ and $x = 2$.

   (b) Are there any values x for which $f'(x) = 0$?

   Solution: The derivative is 0 — i.e., the slope of the tangent line is horizontal — at $x = 0$. This is the only such place.

(c) This particular function f has an interval on which its derivative $f'(x)$ is constant. What is this interval? What does the derivative function look like there? Estimate the slope of $f(x)$ on that interval.

Solution: The derivative is constant where the graph has constant slope — that is, on the interval $(-4, -2)$. The graph of $f(x)$ is a straight line there, and its slope is about -2. The derivative $f'(x)$ on that interval is just a horizontal line at $y = -2$.

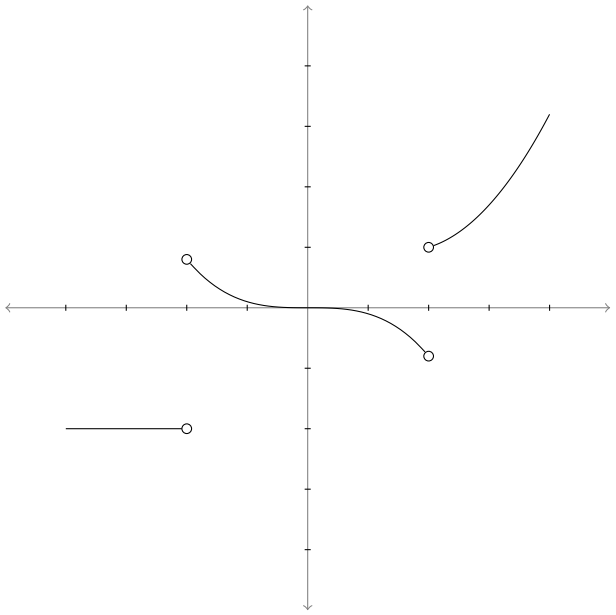
(d) On which interval or intervals is $f'(x)$ positive?

Solution: The derivative $f'(x)$ is positive where f is increasing; that is, on the intervals $(-2, 0)$ and $(2, 4)$.

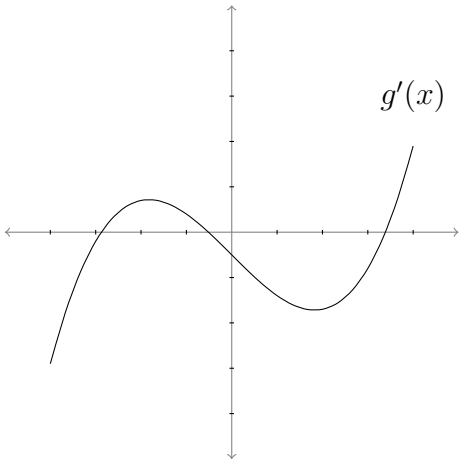
(e) On which interval or intervals is $f'(x)$ negative? Again, sketch a graph of the derivative on those intervals.

Solution: The derivative $f'(x)$ is negative where f is decreasing; that is, on the intervals $(-4, -2)$ and $(0, 2)$.

(f) Now use all your answers to the questions to sketch a graph of the derivative function $f'(x)$ on the coordinate plane below.



2. Below is a graph of a derivative $g'(x)$. Assume this is the entire graph of $g'(x)$. Use the graph to answer the following questions about the original function $g(x)$.



(a) On which interval or intervals is the original function $g(x)$ increasing?

Solution: The function $g(x)$ is increasing on the intervals where $g'(x)$ is positive. From the graph, we see that these intervals are approximately $(-2.9, -0.5)$ and $(3.4, 4)$.

(b) On which interval or intervals is the original function $g(x)$ decreasing?

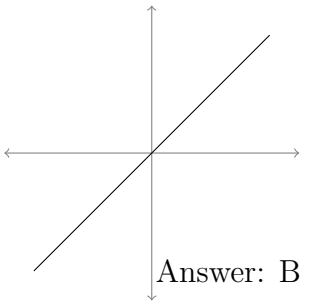
Solution: The function $g(x)$ is decreasing on the intervals where $g'(x)$ is negative. From the graph, we see that these intervals are approximately $(-4, -2.9)$ and $(-0.5, 3.4)$.

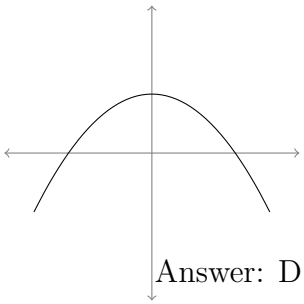
(c) Now suppose $g(0) = 0$. Is the function $g(x)$ ever positive? That is, is there any x so that $g(x) \gneq 0$? How do you know?

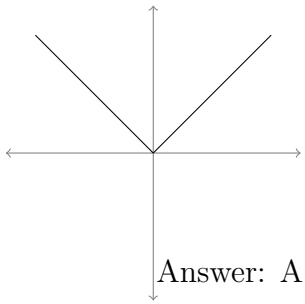
Solution: The graph says that $g'(0) < 0$, which means that $g(x)$ is decreasing at $x = 0$. So the function $g(x)$ is decreasing as it passes through the point $(0, g(0)) = (0, 0)$, which means that it must have been positive shortly before hitting 0. So yes, $g(x)$ is sometimes positive.

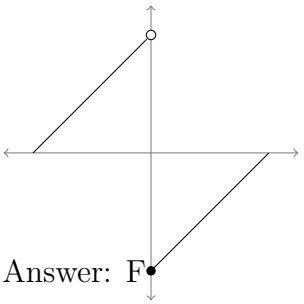
3. Six graphs of functions are below, along with six graphs of derivatives. Match the graph of each function with the graph of its derivative.

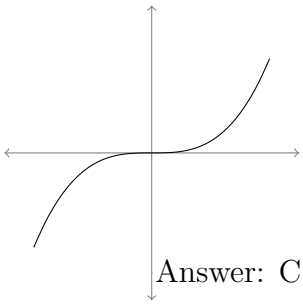
Original Functions:

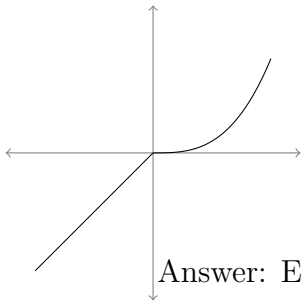

Answer: B


Answer: D


Answer: A


Answer: F


Answer: C


Answer: E

Their derivatives: