

Worksheet for Week 10: Sketching Curves

You might have wondered, *why bother learning how to sketch curves using calculus if I can just plug the equation into a computer and see the graph?* But it could happen that you don't actually have a neat formula for the function you're trying to graph. In this worksheet, you'll reconstruct a graph of a function given some data about it. There is no neat formula for the function, but calculus will help you figure out what it looks like anyway!

At the end of the worksheet, there is a problem about maximizing a function on a closed interval, for extra practice.

1. A scientist is watching a bug walk back and forth along a line. Suppose the line has a coordinate x , and let $p(t)$ be the continuous function giving the bug's position on the line at time t (in seconds).



The scientist observes the bug's motions and records what she sees:

- At $t = 0$, the bug was located at $x = -0.25$.
- At $t = 5$, the bug passed through the point $x = 0$ for the first and only time.
- As $t \rightarrow \infty$, the bug approaches $x = 0$. In other words, $\lim_{t \rightarrow \infty} p(t) = 0$.
- The derivative function $p'(t)$ — the velocity of the bug — is continuous. For a few seconds at the beginning $p'(t)$ was negative, but then it crossed 0 to become positive at $t = 3.3$. It crossed 0 to become negative again at $t = 6.7$, and remained negative thereafter.
- Also, $p'(t)$ had its maximum value at $t = 5$, and its most negative value at $t = 2.2$ and $t = 7.8$.
- The bug always stayed within 5 units of $x = 0$.

In this problem, you'll figure out how to sketch a graph of the bug's movements on the interval $[0, \infty)$, even though you don't know the formula for $p(t)$! For now, use the information above to answer the following questions. (*After* you answer them all, you'll get to make the sketch.)

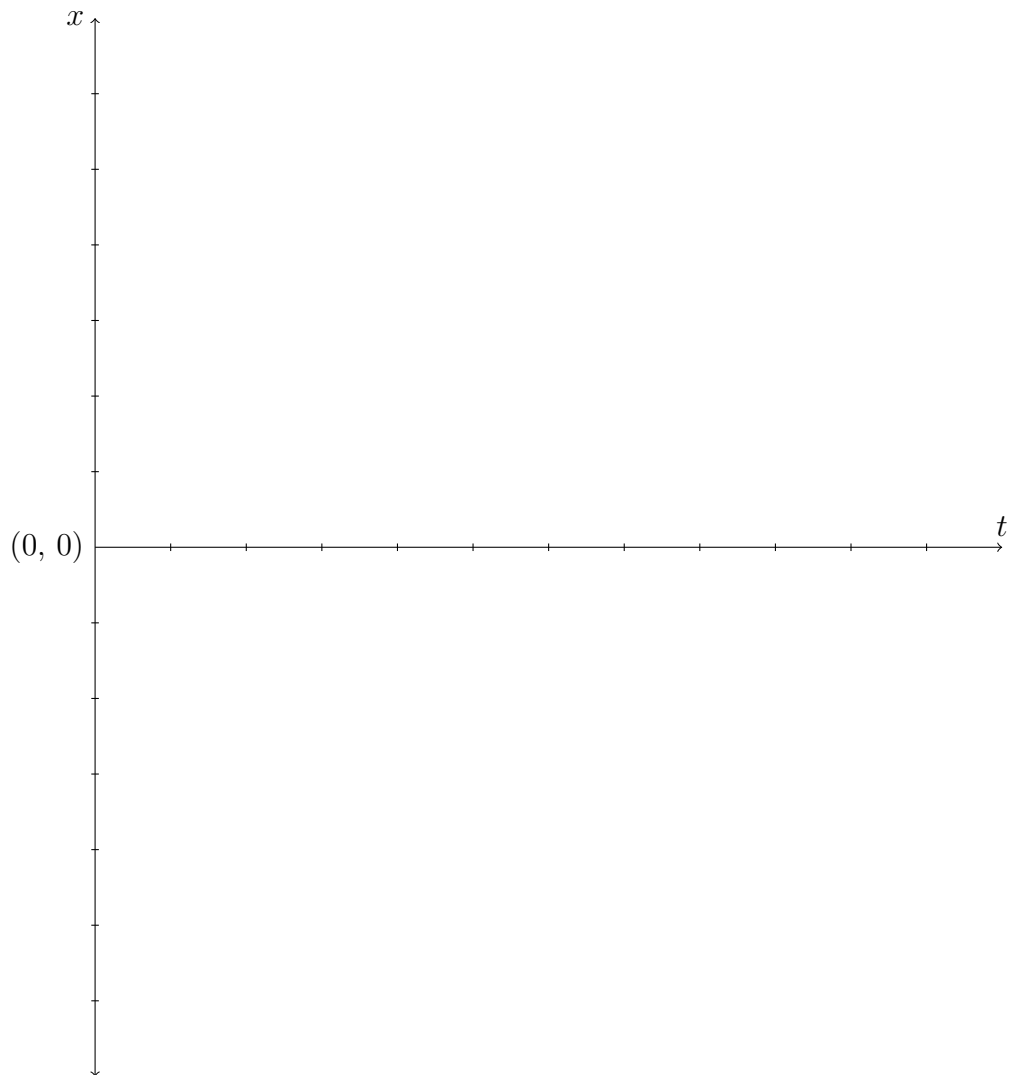
- (a) Where will the graph of $p(t)$ intersect the t and x axes?

(b) Does the graph of $p(t)$ have any asymptotes? If so, where are they?

(c) Where is $p(t)$ increasing and where is it decreasing?

(d) What are the t -coordinates of the local minima and maxima?

- (e) Using all the information above, and your answers to the questions, sketch a graph of $p(t)$ on the plane below.



2. Brooke is located 5 miles out at sea from a straight shoreline in her kayak. She wants to make it to the taco truck on the beach for lunch, which is 6 miles from point A on shore (see picture). Brooke can paddle 2 miles/hour and walk 4 miles/hour. If she paddles along a straight line to shore, find an equation for the total time it will take Brooke to get to lunch. Your equation will depend on where Brooke beaches the boat. Where should she land the boat to eat as soon as possible?

