1. (12 total points) Compute the derivatives of the following functions. Do not simplify your answers.

   (a) (4 points) $f(t) = \tan(3t) \cdot e^{\sqrt{t}}$

$$f'(t) = \sec^2(3t) \cdot 3 \cdot e^{\sqrt{t}} + \tan(3t) \cdot e^{\sqrt{t}} \cdot \frac{1}{2\sqrt{t}}$$

   (b) (4 points) $g(x) = \dfrac{\sin^{-1}(7x)}{2x+1}$

$$g'(x) = \frac{\frac{7}{\sqrt{1-49x^2}} \cdot (2x+1) - 2 \cdot \sin^{-1}(7x)}{(2x+1)^2}$$

   (c) (4 points) $x = \sin\left[1 + \cos\left(1 + \sin(1+t)\right)\right]$

$$\frac{dx}{dt} = -\cos\left[1 + \cos\left(1 + \sin(1+t)\right)\right] \cdot \sin\left(1 + \sin(1+t)\right) \cdot \cos(1+t)$$

2. (10 total points) Consider the curve given by the parametric equations

$$x = t^2 + 6t,\ y = t^3 + 2t^2$$

(a) (5 points) Find the equation of the tangent line to the curve when $t = -1$.

$x(-1) = -5 \qquad y(-1) = 1$

$\frac{dx}{dt} = 2t + 6 \qquad \frac{dy}{dt} = 3t^2 + 4t$

$\left.\frac{dx}{dt}\right|_{t=-1} = 4 \qquad \left.\frac{dy}{dt}\right|_{t=-1} = -1$

$\left.\frac{dy}{dx}\right|_{t=-1} = -\frac{1}{4}$

$y - 1 = -\frac{1}{4}(x+5)$

(b) (5 points) Find all times t when the tangent line has slope 2.

$$\begin{aligned} 2 &= \frac{dy}{dx} \\ &= \frac{3t^2+4t}{2t+6} \\ 4t+12 &= 3t^2+4t \\ 0 &= 3t^2-12 \\ &= 3(t-2)(t+2) \\ t &= -2,\ 2 \end{aligned}$$

3. (10 points) A flashlight is laying on the ground 25 feet from a wall. It is on and pointed straight at the wall. Isobel is walking straight from the wall to the flashlight at a constant speed of 2 feet/second. She is 6 feet tall. How fast is the length of her shadow on the wall increasing when she is 10 feet from the flashlight? Give units in your answer.

Let y be the height of Isobel's shadow on the wall. Let x be her distance from the wall.

Note that $\frac{dx}{dt} = 2$ *and* $x = 15$ *when she is 10 feet from the flashlight.*

We want to compute $\frac{dy}{dt}$ *when* $x = 15$.

By similar triangles,

$$\begin{aligned}
\frac{y}{25} &= \frac{6}{25-x} \\
\frac{1}{25}\cdot\frac{dy}{dt} &= -\frac{6}{(25-x)^2}\cdot\left(-\frac{dx}{dt}\right) \\
\frac{dy}{dt} &= 25\cdot\frac{6}{10^2}\cdot 2 \\
&= 3\ \mathit{feet/second}
\end{aligned}$$

4. (8 points) Compute the equation of the tangent line to the curve $y = (x-1)^x$ at the point where $x = 2$. Use **exact values** only. Do not give decimal approximations.

When $x = 2$, *we compute* $y = (2-1)^2 = 1$.

$$\begin{aligned}
\ln y &= \ln(x-1)^x \\
&= x\cdot\ln(x-1) \\
\frac{1}{y}\cdot\frac{dy}{dx} &= 1\cdot\ln(x-1) + \frac{x}{x-1} \\
\frac{1}{1}\cdot\frac{dy}{dx} &= \ln(2-1) + \frac{2}{2-1} \\
\frac{dy}{dx} &= 2 \\
\\
y-1 &= 2(x-2)
\end{aligned}$$

5. (10 points) Find all the points (a,b) on the curve $2y^3 - 6xy + x^2 = 0$ where the tangent line is vertical.

$$\begin{aligned} 2y^3 - 6xy + x^2 &= 0 \\ 6y^2 \cdot y' - 6 \cdot 1 \cdot y - 6x \cdot y' + 2x &= 0 \\ y' &= \frac{6y - 2x}{6y^2 - 6x} \end{aligned}$$

The tangent line is vertical when $6y^2 - 6x = 0$ *but* $6y - 2x \neq 0$.

To find the points, substitute $x = y^2$ *into* $2y^3 - 6xy + x^2 = 0$.

$$\begin{aligned} 2y^3 - 6 \cdot y^2 \cdot y + \left(y^2\right)^2 &= 0 \\ y^3(y-4) &= 0 \\ y &= 0,\ 4 \end{aligned}$$

But $x = 0$ *when* $y = 0$, *and thus* $6y - 2x = 0$. *The curve is not differentiable at* $(0,0)$.

When $y = 4$, *we compute*

$$\begin{aligned} x &= y^2 \\ &= 16 \end{aligned}$$

The only point is $(16,4)$.