

Math 124, Section G, Winter 2017, Midterm I

January 31, 2017

Name \_\_\_\_\_

TA/Section \_\_\_\_\_

**Instructions.**

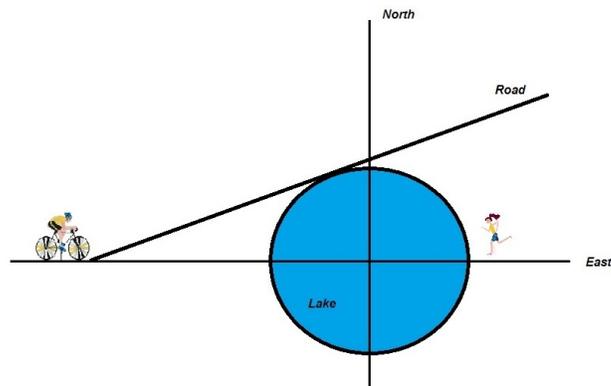
- There are 4 questions. The exam is out of 50 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. **Hand in your notes with your exam paper.**
- You may use a TI-30x IIS calculator. Even if you have a calculator, give me exact answers. ( $\frac{2\ln 3}{\pi}$  is exact, 0.7, 0.70 and 0.699 are approximations for the same number.)
- **Show your work.** If I cannot read or follow your work, I cannot grade it. You may not get full credit for a right answer if your answer is not justified by your work. If you continue at the back of a page, make a note. Please BOX your final answer.

Question	points
1	
2	
3	
4	
Total	

1. (14 points) Anthony and Cleopatra are out on a sunny day. Anthony is riding his bike on a road that is tangent to a small circular lake as shown. We impose a coordinate system where the center of the lake is at the origin. The distance is measured in meters. The picture is not to scale. Anthony's position at time  $t$  in seconds is given by the pair of equations

$$x(t) = -101.4 + 3.6t$$

$$y(t) = 1.5t.$$



- (a) The radius of the lake is 39 meters. Find the coordinates of the point where the road is tangent to the lake.
- (b) How long does it take Anthony to get to that point?
- (c) At  $t = 0$  Cleopatra starts running counterclockwise around the lake, starting at the easternmost point of the lake. How fast is she running, in meters per second, if the two bump into each other during her first lap?

2. (12 points) Differentiate the following functions. Simplify the third one only. Make sure you use parentheses correctly.

$$(a) f(x) = \frac{xe^x}{x^2 + 4x + 7}$$

$$(b) g(x) = \frac{5}{\sqrt{x}} - \frac{2}{5x} + 7x^5$$

$$(c) f(x) = \left(1 - \frac{1}{x}\right) \left(1 + \frac{1}{x}\right)$$

3. (12 points) Evaluate the following limits. Show your reasoning through algebra. You can use your calculator to check your answers, but you cannot use numerical values to justify your answers.

(a)  $\lim_{x \rightarrow \infty} \frac{x + 6}{x^3 + 9}$

(b)  $\lim_{x \rightarrow 4} \frac{x^2 - 3x - 4}{\sqrt{x} - 2}$

(c)  $\lim_{x \rightarrow \infty} \left( \sqrt{4x^2 + x} - 2x \right)$

4. (12 points) Find the equations of the two tangent lines of  $y = \frac{2x - 10}{x - 3}$  that pass through the point  $(6, 6)$ .