Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

- Turn off and stow away all cell phones, pagers, radios, mp3 players, and other similar devices.

- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.

- You can only use a Texas Instruments TI-30X IIS calculator.

- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.

- Place $\boxed{\text{a box around your answer}}$ to each question.

- If you need more room, use the backs of the pages and indicate that you have done so.

- Raise your hand if you have a question.

- This exam has 8 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	13	
2	12	
3	13	
4	13	

Question	Points	Score
5	10	
6	14	
7	12	
8	13	
Total	100	

1. (13 total points) Calculate the derivatives of the following functions. You do not need to simplify your answers.

   (a) (4 points) $f(x) = \dfrac{\cos^{-1}(x)}{\sqrt{1 + x^4}}$

   (b) (4 points) $g(t) = \sin\left[\,2 + \sin\left(2 + \sin(2 + t)\right)\right]$

   (c) (5 points) $y = \left(x^2 + e^{-x}\right)^{10x}$

2. (12 total points) Evaluate the following limits. Show the algebra work where applicable. If the limit does not exist, explain why not.

    (a) (4 points) $\displaystyle\lim_{x\to 1} \frac{3 - \sqrt{4 + 5x}}{x^2 + 2x - 3}$

    (b) (4 points) $\displaystyle\lim_{x\to 3^-} \frac{\frac{1}{|x-3|} - \frac{1}{x-3}}{x - 3}$

   (c) (4 points) $\displaystyle\lim_{x\to 0} \frac{\sin^2(2x)}{3x \sin(5x)}$

3. (13 points) A spherical soap bubble has radius r and thickness h. The volume of soap fluid that forms the bubble is related to the radius and thickness by the formula:

$$V = \frac{4}{3}\pi \left[\left(r + \frac{h}{2}\right)^3 - \left(r - \frac{h}{2}\right)^3 \right]$$

Suppose that the bubble is expanding but the total volume of soap fluid remains constant. At a certain moment, the radius is 20 cm and increasing at 0.5 cm/sec. At the same time, the thickness is 2 cm[1]. How fast is the thickness decreasing at this moment?

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[1]The bubble thickness is chosen to make the calculations simpler. It is not realistic

4. (13 points) A cylindrical can of volume 250 cubic centimeters is to be made from aluminum. The side is made from a thin sheet which costs 0.2 cents per square centimeter. The top and bottom of the can is made from a thicker sheet which costs 0.4 cents per square centimeter. Moreover, since the top and the bottom are circles, they have to be cut from square pieces. The wasted area between the circle and the square can be sold back to the aluminum supplier at a price of 0.1 cent per square centimeter to be recycled.

SIDE

$2\pi r$

TOP AND BOTTOM

h

r
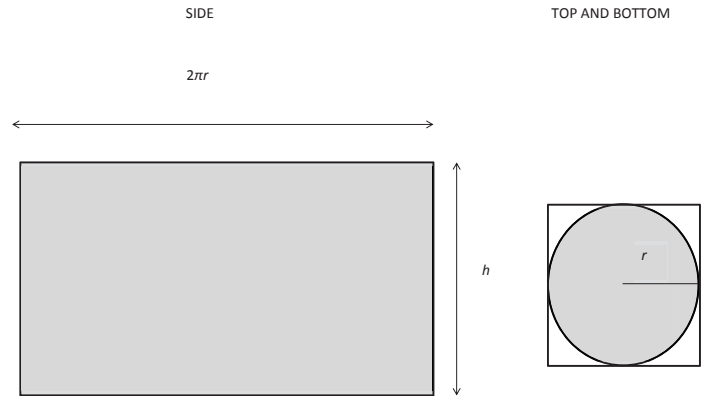
What are the radius r and height h of the minimal cost can? Verify that your answer is really a minimum.

Give your answer in exact form and as a decimal approximation. Note: Given a cylinder of radius r and height h, the volume is $V = \pi r^2 h$.

5. (10 points) Find the equation of the tangent line to the curve defined by

$$x^2 y + xe^y = 2$$

at the point $(2, 0)$. Use it to approximate the value of y when $x = 2.2$.

6. (14 total points) Answer the following questions about the function $f(x)$ whose graph on the interval $[-5, 5]$ is shown.


$y = f(x)$
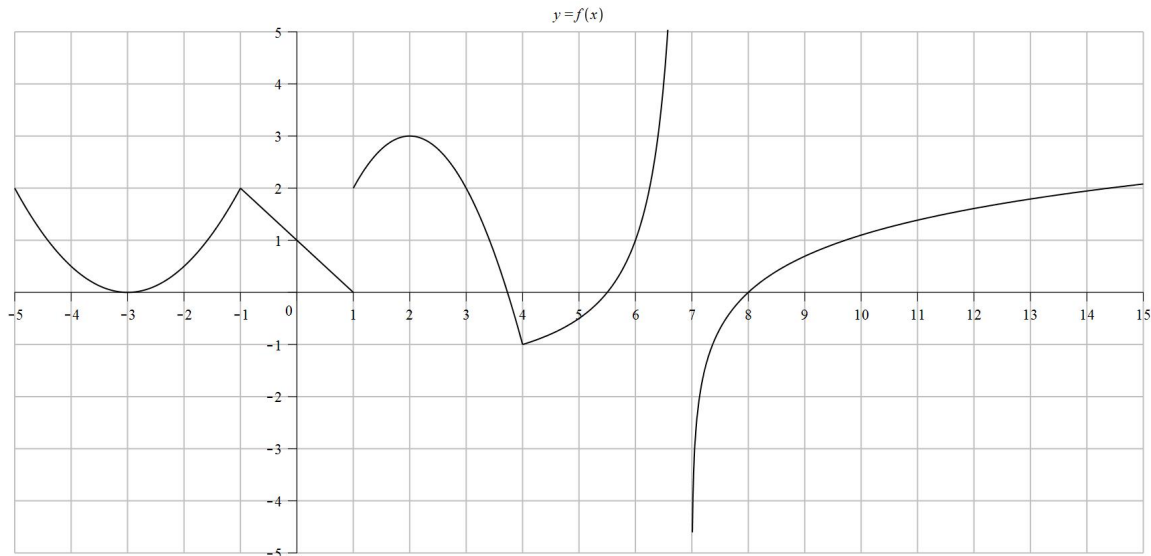
(a) (2 points) List the value(s) of x where the function is not differentiable.

(b) (2 points) List the value(s) of x where the function is not continuous.

(c) (2 points) List the interval(s) where $f'(x)$ is positive.

(d) (2 points) List the interval(s) where $f'(x)$ is decreasing.

(e) (2 points) $\lim\limits_{x \to 7^+} f(x) =$

(f) (2 points) List the following from the least to the greatest: $f'(13)$, $f(6)$, $f'(2)$, $f''(2)$

(g) (2 points) $\lim\limits_{h \to 0} \dfrac{f(2+h) - 3}{h} =$

7. (12 total points) The motion of a point on a bicycle wheel is described by the parametric equations below. A and B are positive constants. Time is measured in seconds and distance in meters (its a big wheel).

$$
\begin{aligned}
x(t) &= At + \cos(2t) \\
y(t) &= B - \sin(2t)
\end{aligned}
$$
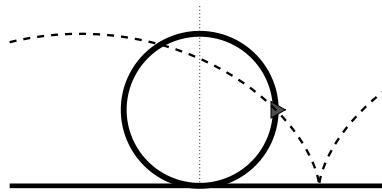
(a) (3 points) How long does it take for the wheel to make one revolution?

(b) (3 points) Use the fact that the y coordinate of the point is zero when it is at the bottom of the wheel to find the constant B.

(c) (3 points) Use the fact that the horizontal velocity of the point is zero when it is at the bottom of the wheel to find the constant A.

(d) (3 points) Find the speed of the point when it is at the top of the wheel. *Hint: If you couldn't do parts (b) or (c) you may include the letters A and/or B in your answer .*

8. (13 total points)Let $f(x) = \dfrac{2x}{\sqrt{1+x^2}}$. Answer the following questions, showing your reasoning.

   (a) (3 points) List all horizontal and vertical asymptotes of $y = f(x)$.

   (b) (5 points) Find any critical numbers for $f(x)$ and specify if they correspond to a local minimum, a local maximum, or neither.

   (c) (5 points) Find the inflection points of $f(x)$ and specify the intervals on which the function is concave up, respectively down.