## Math 124

## **Final Examination**

Your Name	Your Signature	
Student ID #	Quiz Section	
Professor's Name	TA's Name	

- Turn off and stow away all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one  $8.5'' \times 11''$  sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 8 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	15	
2	15	
3	12	
4	13	

Question	Points	Score
5	13	
6	20	
7	12	
Total	100	

1. (15 total points) Find the derivatives of the following functions. Do not simplify your answers.

(a) (5 points) 
$$f(x) = \frac{\tan x}{\sqrt{1 + e^{x^2}}}$$

(b) (5 points)  $y = (\tan x)^{x^2} \cdot (x+1)^{2/3}$ 

(c) (5 points) 
$$g(t) = \ln\left(\left(1 + \frac{t}{\ln t}\right)^4\right)$$

2. (15 total points) Find the following limits. In each case your answer should be either a number,  $+\infty$ ,  $-\infty$ , or DNE. Please show your work.

(a) (5 points) 
$$\lim_{x \to 2} \frac{e^{x^2} - e^4}{x - 2}$$

(b) (5 points) 
$$\lim_{t \to 0} \left( \frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$$

(c) (5 points) 
$$\lim_{x \to 0} \frac{\sin(5x) \sin(3x)}{x \sin(2x)}$$

- 3. (12 total points) The point (2, 1) lies on the curve  $x^3 + x^2y + y^3 = 13$ .
  - (a) (4 points) Find the equation of the tangent line to the curve at this point.

(b) (4 points) Use this tangent line to approximate the y-coordinate of a point on the curve where x = 1.92. Give your answer to three decimal places.

(c) (4 points) Is your answer in (b) an over-estimate or an under-estimate?

4. (13 points) A highway patrol plane flies 0.8 miles above a straight road at a steady gound speed of 120 miles per hour. The pilot sees an oncoming car and, with radar, determines that the line of sight distance from plane to car is 1.7 miles, decreasing at a rate of 160 miles per hour. Calculate the car's speed along the highway.

5. (13 points) A manufacturer needs to make a cylindrical can that will hold 1.5 liters of liquid. Determine the dimensions of the can (base radius and the height) that will minimize the amount of material used in its construction. Recall that 1 liter = 1000cm<sup>3</sup>.

Be sure to verify that your answer is a minimum.

- 6. (20 total points) Consider the function  $f(x) = x^5 7x^4 + 12x^3$ .
  - (a) (4 points) Find the x and y intercepts of the graph of y = f(x).

(b) (6 points) Find the critical numbers for f(x) and determine if each gives a local minimum, a local maximum, or neither.

- 6. (continued) Recall the function  $f(x) = x^5 7x^4 + 12x^3$ 
  - (c) (6 points) Find the inflection point(s) of y = f(x). In which interval(s) is the graph concave down?

(d) (4 points) Sketch the graph of y = f(x) on the axes provided below. Mark the coordinates of any local maximum, local minimum or inflection point. Make sure your picture matches the information you provided in parts (a)-(c).



- 7. (12 total points) Consider the function  $g(x) = \begin{cases} 3x^2 + 4x & \text{if } x \leq 1 \\ 2x^3 + cx + d & \text{if } x > 1 \end{cases}$  where c and d are constants.
  - (a) (8 points) Find the values c and d so that g(x) is differentiable at x = 1.

(b) (4 points) Let c and d be your answer from part (a). Is the **derivative** g'(x) continuous at x = 1?