PLEASE NOTE: If you are not yet enrolled but intend to enroll when space opens up, be sure to attend all classes and sections and do the assigned work, especially the conceptual problems. No student will be excused from the first week’s work.

WINTER 2020 — MATH 308 SECTIONS A AND B

Welcome to Math 308! Linear algebra is of central importance in real-world applications of mathematics, at least as much as calculus. It is not unusual to need to solve systems of equations in hundreds of variables — although in this course we will not be using computers and so will limit ourselves to a small number of unknowns.

Most of the information you need about the course, including a week-by-week syllabus, can be found at the general course website: www.math.washington.edu/~m308. This information sheet concerns sections A and B specifically and does not necessarily apply to other sections.

Please attend all lectures. You are responsible for all announcements, as well as all examples, given in lecture. The exams will be based mainly on the conceptual problems and the in-class examples. You should also attend all Tuesday sections, which will be devoted to student presentations of the conceptual problems. Each student is required to give one presentation during the quarter (see below). Please come on time to class, so that you don’t miss announcements and material presented at the beginning. If you miss class or are late to class for any reason, it is your responsibility to find out what you missed from a classmate. If you miss a midterm (for example, because of sickness) please inform us right away; in that case you’ll be able to take a make-up exam roughly a week later.

Please do not have a computer in use during lecture. Research has shown that computer screens are distracting to other students. Cellphones also should be off during lecture, except that it’s okay to use them briefly to photograph the screen. You will not need to have the textbook with you during lecture or section.

WEBASSIGN: The course webpage has instructions for getting access to the webassign homework. Note that you must login at the UW portal, in which case you do not need a class code. In case of difficulties that you cannot get resolved by contacting webassign customer service, you can get them cleared up by a webassign representative, who will have office hours early in the quarter (details will be announced in lecture).

CONCEPTUAL PROBLEMS: The most important part of the homework is the conceptual questions (linked to from the week-to-week syllabus on the course webpage). You should do them carefully — exams will be based on the conceptual problems more than the webassign problems. The conceptual problems will be due during Tuesday section. They will not be corrected (although they might be spot-checked); rather, each assignment will get 0 to 3 points based on completeness. The conceptual problems will also be the topic of Tuesday’s student presentations.

GRADING: There is no dropped exam or homework. A 90% score counts as 4.0; the conversion from percent to a grade proceeds linearly, with 70% being a 3.0, 50% being a
2.0, and so on. In computing the course grade there will be a maximum of 250 points total: 50 for each midterm, 100 for the final exam, and 50 for everything else. Those last 50 points are distributed as follows. The webassign total counts for 10 points (for example, 85% of the maximum possible webassign points translates to 8.5 points), the conceptual problems handed in count for 10 points, and the oral presentation of a conceptual problem counts for 30 points (that is, 12% of the course grade).

GOOD EXAM-TAKING STRATEGY: (1) First solve the problems that you are confident about (these are not necessarily the first ones and they are not necessarily the same ones for all students), taking your time, checking your work as you go along, and avoiding careless mistakes. When you are sure of those solutions, then proceed to the problems you find more difficult. (2) Read a problem carefully and think about it before starting work. Usually there is no partial credit if a problem is set up wrong, or if the initial stages in solving the problem are incorrect. (3) Show your work clearly and legibly. (4) If any of your work is out of sequence, show by arrows the order in which your solution needs to be read. (5) Box your final answers. (8) Don’t leave an exam early unless you have carefully checked all your work.

CHEATING: Any talking to a friend, looking on another student’s paper, or use of a cellphone (or any other unauthorized device) during an exam constitutes cheating. If you observe any instance of cheating, please report it to me or your T.A. immediately with as many details as possible so that I can take appropriate action. It is unfair to honest students to allow anyone to get a higher grade by cheating.

HOMEWORK GIVEN ON JAN 6 TO BE HANDED IN ON TUES JAN 7

1. (This is #2 in the conceptual problems for Chapter 1.) Before paying employee bonuses and state and federal taxes, a company earns profits of $103,000. The company pays employees a bonus equal to 5% of after-tax profits. State tax is 5% of profits (after bonuses are paid). Finally, federal tax is 40% of profits (after bonuses and state tax are paid). Calculate the amounts paid in bonuses, state tax and federal tax.

2. (This is #6 in the additional list of conceptual problems.) The circle \(x^2 + y^2 = a + bx + cy\) passes through \((10, -2)\) and \((-5, 3)\), and its tangent line at \((-5, 3)\) has slope 8. Find \(a, b, c\), and also the center and radius of the circle.

PRESENTATIONS OF CONCEPTUAL PROBLEMS

1. Volunteer AFTER doing the problem you’re volunteering for and being confident that you can give a nice, conceptual explanation. Never volunteer to present a problem if you’re confused about the concepts that the problem illustrates.

2. The presentation has to go smoothly and efficiently, since time is very limited, and it must not get bogged down in routine computations. In fact, some computations, such as row operations, can be omitted with only the results written on the board. If you take too much time, the T.A. can end the presentation before it’s finished.
3. A presentation must explain concepts, not just calculations. It must be clear what the point of the problem is.

4. During a presentation the TA or other students might interrupt when something is confusing or in error. This is normal, and should not be taken badly. The TA or other students might also ask for more information about strategy for solving a problem (how did you know to take that step? is there any easier, or alternative, way you could have done that? is there a geometrical explanation for what you just did? can you explain that step in words rather than symbols?).

5. It is best not to use a computer (e.g., PowerPoint) for your presentation, since that can cause delays and other problems. You might want to use the doc cam if you intend to have a lot written down. Otherwise it’s fine to use the whiteboard.

6. If a presenter is unprepared and flounders, wasting everyone’s time, the T.A. will tell the presenter to sit down and, in an extreme case, give 0.0 for the presentation.

7. There’s only one chance. No make-ups for a bad presentation.

8. Presentations need to be carefully prepared. It’s a good idea to practice your presentation with a friend or classmate in advance.

9. Presentations will be graded primarily on how clear and helpful they are to your classmates. If you aren’t sure you can give a good presentation of a challenging problem, you should volunteer to do one of the more routine assigned conceptual problems. But a good presentation of a routine problem will not be graded as generously as a good presentation of a challenging problem.

10. 12% of the course grade will be based on your presentation (along with some credit if you make helpful comments or questions during other presentations). This is three times as much as all the webassign together (4%).

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Math 308  
SUPPLEMENTARY CONCEPTUAL PROBLEMS  
Neal Koblitz

**Please note:** Expect to find problems similar to the ones below on the midterms and final exam. The assigned conceptual problems from the problem sets on the syllabus (on the course webpage) are also similar to the types of problems you will find on the exams. Except for #6, in the problems below solve the systems of linear equations by setting up augmented matrices and performing row operations to get them in reduced echelon form. In the case of #6, which is due the first Tuesday of the quarter, you can solve the equations by adding/subtracting multiples of one equation from another so as to eliminate variables.

1. A hyperplane through the origin in \( \mathbb{R}^5 \) has an equation of the form \( ax_1 + bx_2 + cx_3 + dx_4 + ex_5 = 0 \). Find the hyperplane that contains the following four vectors:

\[
\begin{bmatrix}
1 \\
0 \\
1 \\
1 \\
0
\end{bmatrix}, \quad \begin{bmatrix}
0 \\
1 \\
0 \\
1 \\
1
\end{bmatrix}, \quad \begin{bmatrix}
1 \\
1 \\
0 \\
1 \\
1
\end{bmatrix}, \quad \begin{bmatrix}
1 \\
1 \\
0 \\
1 \\
1
\end{bmatrix}
\]
2. The cubic \( y = a + bx + cx^2 + x^3 \) crosses the \( x \)-axis when \( x = 1 \), has a maximum when \( x = 2 \), and has a point of inflection when \( x = 3 \). Find the coefficients \( a, b, c \). First set up equations they satisfy, then write out the system’s augmented matrix, and reduce it to reduced echelon form.

3. The hyperbola \( a + bx + x^2 = cy + y^2 \) passes through the points \((6, 7)\) and \((4, 1)\), and at the point \((4, 1)\) its tangent line has slope \(-5\). Find \( a, b, c \). (You’ll need to use implicit differentiation.)

4. The function \( f(t) = (a + bt + ct^2)e^t \) passes through the points \((1, 4e)\) and \((2, 9e^2)\). At \((1, 4e)\) its tangent line has slope \(7e\). Find the coefficients \( a, b, c \).

5. (This is an old midterm problem.) An elliptic curve (not the same as an ellipse) is a curve with equation of the form \( y^2 = a + bx + cx^2 + x^3 \). Suppose that the elliptic curve passes through the point \((1, 2)\), where it has derivative \( \frac{dy}{dx} = \frac{5}{4} \), and also through the point \((2, 3)\).

(a) Find three linear equations for the three unknowns \( a, b, c \). (You’ll have to use implicit differentiation to get one of them.) Arrange the equations so that the unknowns are on the left and constants are on the right of the equal signs.

(b) Write the augmented matrix of the system of linear equations in part (a), and use row operations to get it to reduced echelon form.

(c) Write out the equation of the elliptic curve.

6. The circle \( x^2 + y^2 = a + bx + cy \) passes through \((10, -2)\) and \((-5, 3)\), and its tangent line at \((-5, 3)\) has slope \(8\). Find \( a, b, c \), and also the center and radius of the circle.

7. Sinusoidal functions are a basic building block for modeling periodic phenomena (things that repeat at regular intervals). For background please see the “sinusoidal functions supplement” linked to from Week 4 of the week-by-week syllabus on the Math 124 webpage (which has the same url as Math 308 except with 308 replaced by 124). Suppose we want to use a sinusoidal function to give a (rough) formula for the way the temperature \( T \) in degrees Celsius varies with time \( t \) (hour of the day on the 24-hour clock) at a certain time of the year. That function will have the form

\[
T = f(t) = A \sin \left( \frac{2\pi}{24}(t - C) \right) + D,
\]

where \( A \) is the amplitude, \( D \) is the vertical shift (in other words, \( T = D \) is the horizontal line around which the temperature oscillates), and \( C \) is the phase shift. At first \( A, C, D \) are unknown to you. But suppose we know the average temperature at three different times of day. That is, suppose that the sinusoid curve in the \( tT \)-plane passes through the three points \((8, 4)\), \((12, 10)\), \((16, 13)\). (Note: On the 24-hour clock, 12 is noon and 16 is 4 pm.)

(a) The three points give three equations with three unknowns \( A, C, D \), but they are not linear equations. Change to new variables defined by: \( \alpha = D \), \( \beta = A \cos \left( \frac{2\pi}{24}C \right) \), \( \gamma = A \sin \left( \frac{2\pi}{24}C \right) \). Note that after you find \( \alpha, \beta, \gamma \) you can quickly find \( A, C, D \), because
\[ D = \alpha, \ A = \sqrt{\beta^2 + \gamma^2}, \text{ and } \frac{2\pi}{24} C = \arctan(\gamma/\beta). \] Using the trig identity for \(\sin(X - Y)\), write three equations in terms of the unknowns \(\alpha, \beta, \gamma\).

(b) Write the augmented matrix for this system of equations, transform it to reduced echelon form, and find \(\alpha, \beta, \gamma\).

(c) Write the formula for \(T = f(t)\). Be careful (by checking the value at one of the points) about deciding which of two possible values for \(C\) is correct.

8. In each of the following, \(\bar{x}\) is the vector whose components are the coefficients in \(f(t)\), and \(\bar{y}\) is the vector whose components are the coefficients in \(f'(t)\). Taking the derivative of \(f(t)\) in the usual way, you can find formulas for the \(y\)'s in terms of the \(x\)'s. Then find a matrix \(A\) such that \(\bar{y} = A\bar{x}\).

(a) \(f(t) = x_1 + x_2t + x_3t^2 + x_4t^3\), \(f'(t) = y_1 + y_2t + y_3t^2 + y_4t^3\) (the matrix \(A\) will have a lot of zeros).

(b) \(f(t) = x_1 \cos(\omega t) + x_2 \sin(\omega t)\), \(f'(t) = y_1 \cos(\omega t) + y_2 \sin(\omega t)\) (where \(\omega\) is a constant, called the angular frequency).

(c) \(f(t) = x_1 e^{-kt} \cos(\omega t) + x_2 e^{-kt} \sin(\omega t)\), and \(f'(t)\) has the same form with coefficients \(y_1, y_2\). (The function \(f(t)\) is called damped harmonic oscillation.)

(d) \(f(t) = x_1 \sin(t) + x_2 \cos(t) + x_3 t \sin(t) + x_4 t \cos(t)\), and \(f'(t)\) has the same form with coefficients \(y_1, y_2, y_3, y_4\).

(e) \(f(t) = (x_1 + x_2t + x_3t^2)e^{-kt}\), and \(f'(t)\) has the same form with coefficients \(y_1, y_2, y_3\).

9. To go from a geometrical description of a linear transformation \(\mathbb{R}^2 \to \mathbb{R}^2\) to its matrix \(A\), you have to ask two questions: What does it do to the first standard basis vector \([1, 0]^T\)? The answer to that question is the first column of \(A\). To get the second column, ask: What does the transformation do to the second standard basis vector \([0, 1]^T\)? In parts (a)–(c), find the matrix of the linear transformation that's described:

(a) Reflects about the \(x_2\)-axis and then rotates counterclockwise through 90°.

(b) Rotates counterclockwise through 30°, then reflects about the line \(x_2 = x_1\), and finally expands all vectors by a factor of 2.

(c) Expands vectors horizontally by a factor of 2 (that is, doubles their \(x_1\)-coordinates), then rotates clockwise 45°, and then again expands vectors horizontally by a factor of 2.