## Education Reform for the Modern World

Pretty much everywhere, education has trouble keeping up with the demands of modern society. Vietnam is no exception. I will discuss some ways to address this problem that are realistic and not very expensive. I will start with mathematics education, and then talk more generally about preparing students for a professional life in the hightech world.

Traditionally, in Vietnam as in other countries, mathematics has been taught in a formalistic way. The idea is to develop skill in arithmetic computation, algebra, trigonometry, and so on. Applications of mathematics for the most part have to wait until much later, perhaps in a physics or engineering program at the university level.

A problem with this approach - especially in countries such as Vietnam where admission to a good university depends on a high score on a difficult math exam - is that most young people learn to view mathematics as an obstacle they must overcome in order to go to a good university. Once they're admitted, they can forget about mathematics, or, at the very most, take a few required math courses and then forget about mathematics. They see mathematics as detached from what's important to them - with the lone exception of university admission.

This is shortsighted. Mathematics is not only a set of computational techniques. It is also a way to look at the world and to reason about it. From the early primary grades through the first year of university studies, there are many ways students can start relating mathematics to the world they live in.

Classes in the primary grades should include interludes for recreational mathematics. For example, in visiting schools (including a few in Hanoi),

I have often introduced a game using five dice. The object, using arithmetic operations (and taking squares of numbers and expressions), is to construct a large prime number. The children divide into teams and try to get the largest prime in the class.

Many games and puzzles that are mathematical in nature are simple enough for young children. Some involve cryptography - secret communications - using shifts and other simple transformations of individual letters or blocks of letters.

At the secondary level more realistic applications of mathematics can be woven into the curriculum. Students can begin to learn how to translate a "word problem" (also called a "story problem") from words into mathematics. By a "word" or "story" problem I do not mean simply a problem with a lot of words or a story. I mean a problem that requires thought and understanding to translate from the words into mathematics. From what I've seen, the current Grade 12 textbooks and standardized exams in Vietnam contain hardly any problems of that type.

A simple algebra problem of this sort might be to calculate when two bicyclists traveling toward one another will meet, given their speeds and their distance apart. A more difficult example would be to write parametric equations for a point on the rim of a wheel that's spinning as the wheel follows a parabolic trajectory on an airless planet - given the speed and angle at which the wheel is thrown, its rate of rotation, its radius, the initial location of the point, and the planet's gravitational acceleration at the surface. Another challenging problem asks the student to use linear algebra to fit a sinusoidal curve

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y=A \sin ((2 \pi / 24)(t-C))+B
$$

to three given data points for the average temperature at time $t$ on a summer day. Faced with equations that seem highly nonlinear, the student has to transform them to a form where linear algebra can be used.

Students should have some exposure to the use of math and statistics in the non-sciences. Here is one example I've given to students in the middle and secondary grades. I give them a list of wages of workers, supervisors, managers, and the owner in a small factory. I ask the children to compute (1) the mean of the wages and (2) the median. I then ask which would be used by the workers who write a leaflet explaining why they are going on strike for an increase in wages, and which would be used by the owner to explain why he's refusing to give them an increase.

For example, suppose that for every 1000 USD that goes to pay the people in the factory, 50 USD goes to each of seven workers, 100 USD goes to each of two managers, and the remaining 450 USD goes to the owner. Then the mean is 100 USD, which is much more than most people receive; the median is 50 USD. More generally, the incomes of the very rich inflate the mean so that it seems that almost everyone is better off than they really are.

I like this example partly because it is closely related to a fundamental controversy in real-world economics. Simple though it is, this example could be my way (as a mathematician) of explaining how my philosophical views are closer to socialism than to capitalism. A capitalist analysis of a country's wellbeing typically is based on the mean wealth, that is, gross domestic product divided by the population (GDP per capita). That's a factory owner's point of view. I think that this is a poor measure, and that a better measure would be the living
conditions of the person in the middle, which, roughly speaking, is the median wealth.

The first-year university course in analysis (called Calculus 1 or Analysis 1) should make room for engaging story problems that use basic techniques. Some of my favorite examples use the simplest types of differential equations, namely, small modifications of the differential equation for exponential growth. One such differential equation provides a model for predicting the spread of a disease. Although it's too simple to be accurate for a human epidemic, the graph of the solution gives a good qualitative picture of what happens and vividly displays the inflection point, where the rate of infection starts to decline.

If in the differential equation $\mathrm{d} y / \mathrm{d} t=k y(t)$ for exponential growth we replace the right-hand side by $k y(t)+c$, where $k$ and $c$ are constants, it is still easy to solve, and it can now be used to describe (often quite accurately) a wide variety of processes: (1) falling objects before and after a parachute opens; (2) Newton's law of cooling; (3) the changing amount of pollutant in a lake with incoming and outgoing water flows; (4) long-term payment of a loan with interest. I like this topic because it shows the power of mathematics in our world - a single simple differential equation that can help us understand so many different things.

When I wrote that the curriculum "should make room for engaging story problems," I meant that we should reduce the number of topics so that each one can be covered in depth with a focus on applications. There is no need for students to be drilled on complicated techniques involving algebraic or trigonometric identities - which even scientific researchers in such fields as mathematics, physics, and computer science almost never use. The time spent on tricky calculations, often in
preparation for the university entrance exams, should instead be devoted to learning more useful and interesting things.

Change should be made gradually, so that textbooks and exams can be changed and, most importantly, teachers can fully understand the new material and get used to teaching it. This takes time.

Teachers anywhere, even very good ones, are likely to have difficulty teaching a large amount of new, unfamiliar material. For this reason it is best to start either with application problems that are based on such subjects as algebra, calculus, and linear algebra that are already a standard part of what they teach, or else with problems that, while using mathematics such as graph theory, statistics, or probabilities that are not routinely taught in K-12 or $1^{\text {st }}$-year university math courses, will strike the teachers as rather easy and intuitive. A good source for the latter types of problems is the textbook For All Practical Purposes, published by a consortium of American mathematical associations.

Ultimately, the math curriculum must adapt to meet the needs of our students in the $21^{\text {st }}$ century. Few of them will become pure mathematicians, but many of them will become applied mathematicians, computer scientists, or data scientists - who also need to be well-educated in mathematics. At the University of Washington, where I teach, we have a very popular undergraduate major called Applied and Computational Mathematical Sciences (ACMS). The ACMS program is jointly administered by the Departments of Mathematics, Applied Mathematics, Computer Science, and Statistics.

About 50 years ago, before I came to the University of Washington, there was tension and conflict between the pure and applied mathematicians, which led to the formation of a separate Department of Applied Mathematics. That history is not something we're proud of.

Fortunately, for many years now the Mathematics Department has included several applied mathematicians (such as me), and we have extensive ties with other departments in the mathematical sciences. If mathematicians had stuck with an attitude of snobbism about "pure" mathematics and had continued to look down upon colleagues who work on applied questions, our department would now be a lot less successful in attracting students than it is, a lot smaller than it is, and not nearly as vibrant and productive.

Turning now to the broader question of educating science and technology majors, the best universities should be expected to give them a comprehensive, multidisciplinary education that includes some study of non-sciences. In the U.S. we call this a "liberal arts" education, and at least our high-quality universities have this. For example, students at MIT are required to take eight semester courses in the humanities, arts, and social sciences. The MIT website states: "The requirement enables students to deepen their knowledge in a variety of cultural and disciplinary areas and encourages the development of sensibilities and skills vital to an effective and satisfying life as an individual, a professional, and a member of society." Of course, the efforts of U.S. universities to impart a liberal arts education are not always successful. Some students resist this, and later become technical workers with little knowledge outside their specialty.

During the Covid-19 pandemic we saw some dramatic failures that were due to narrowly trained technical people who were ignorant of the human dimensions of the problems they were trying to solve. I'll give two examples. In March 2020 an institute at my university, staffed mainly by applied mathematicians, predicted a mild Covid epidemic in the U.S. They said that their mathematical methods (a type of "curvefitting" based on the data from Wuhan, China) showed that infections would drop to zero in three months, with a total of about 60,000
deaths. Although broadly trained epidemiologists elsewhere had much less optimistic predictions for the U.S., President Trump in his press conferences seized on the prediction coming from the University of Washington to claim that Covid-19 would be no worse than a bad influenza season and to justify his utterly inadequate response to the pandemic. In reality, the pandemic in the U.S. has lasted four years, with over a million Americans dead. Our per capita death rate is about seven times that of Vietnam.

The wildly wrong prediction of 60,000 deaths was due to the rather foolish assumption that U.S. deaths would follow the same pattern as Wuhan's. The mathematicians implicitly assumed that the response to Covid by the public and by the authorities would be the same in the U.S. as in China. The researchers seemed to be ignorant of the vast cultural, social, and political differences between the two countries.

That was not the only misstep during the pandemic that resulted from a narrowly technical approach that ignored the human side of a problem. Despite a fair amount of hype about contact-tracing apps that would supposedly relieve the burden of contact-tracing by humans, there were hardly any countries where contact-tracing apps achieved high rates of use. Low adoption rates meant they were nearly useless. The main problem in most places was that the developers, who tended to be narrowly trained computer scientists, had underestimated both the frequency and the consequences of false positives and had put little effort into reducing them. The researchers themselves lived in comfortable, spacious surroundings, and didn't seem to realize that most workers do not. False positives are far more common among people who live in high-density housing in close proximity to their neighbors, especially since Bluetooth signals penetrate walls and report contagion between neighbors even when there was no exposure. The consequences of a false positive can be a lot more serious in the
working class than among professionals, because professionals can usually miss work or arrange to work from home without losing pay or suffering reprisals at work. Many in the working class are not so fortunate. But the researchers who developed the apps must have been unconsciously thinking that everyone lives the way they do. A course or two in sociology when they were students hopefully would have taught them otherwise.

I'll conclude with a suggestion for increasing young people's interest in careers in scientific fields. Many scientists have found that outreach to the public and especially to schoolchildren is enjoyable and stimulating. From the primary grades through the first year of university studies, young people are impressionable, and they can be influenced by contact with scientists who are enthusiastic about their work.

Outreach to young people should be encouraged and incentivized. For example, it should be taken into account when evaluating scientists for a promotion or salary raise. They could be released from some other duties if they are devoting a lot of time and effort to visiting schools or talking with beginning university students. Their place of work could give awards to scientists who regularly visit schools and lead the children in lively activities that convey the joys of scientific work.

