NWMI Spring Workshop 2013

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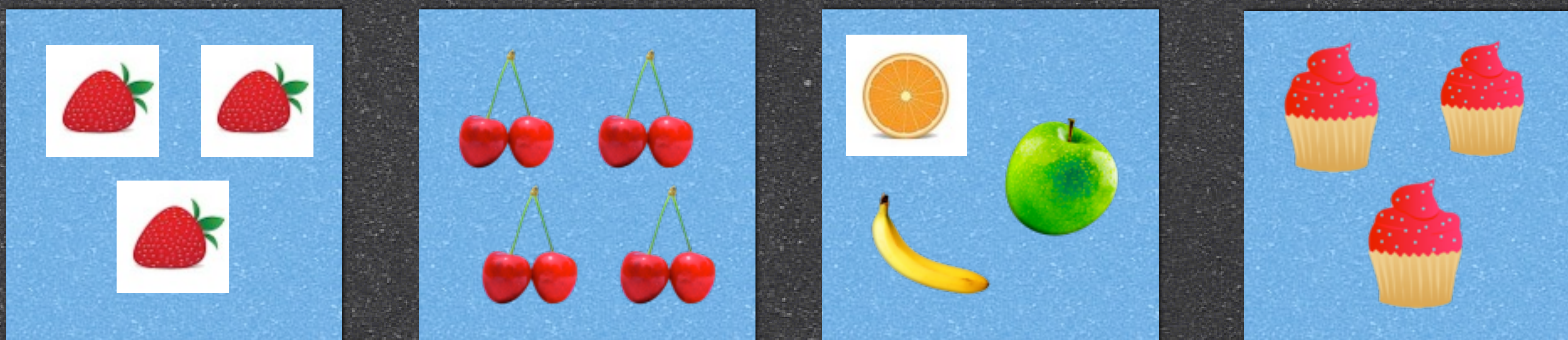
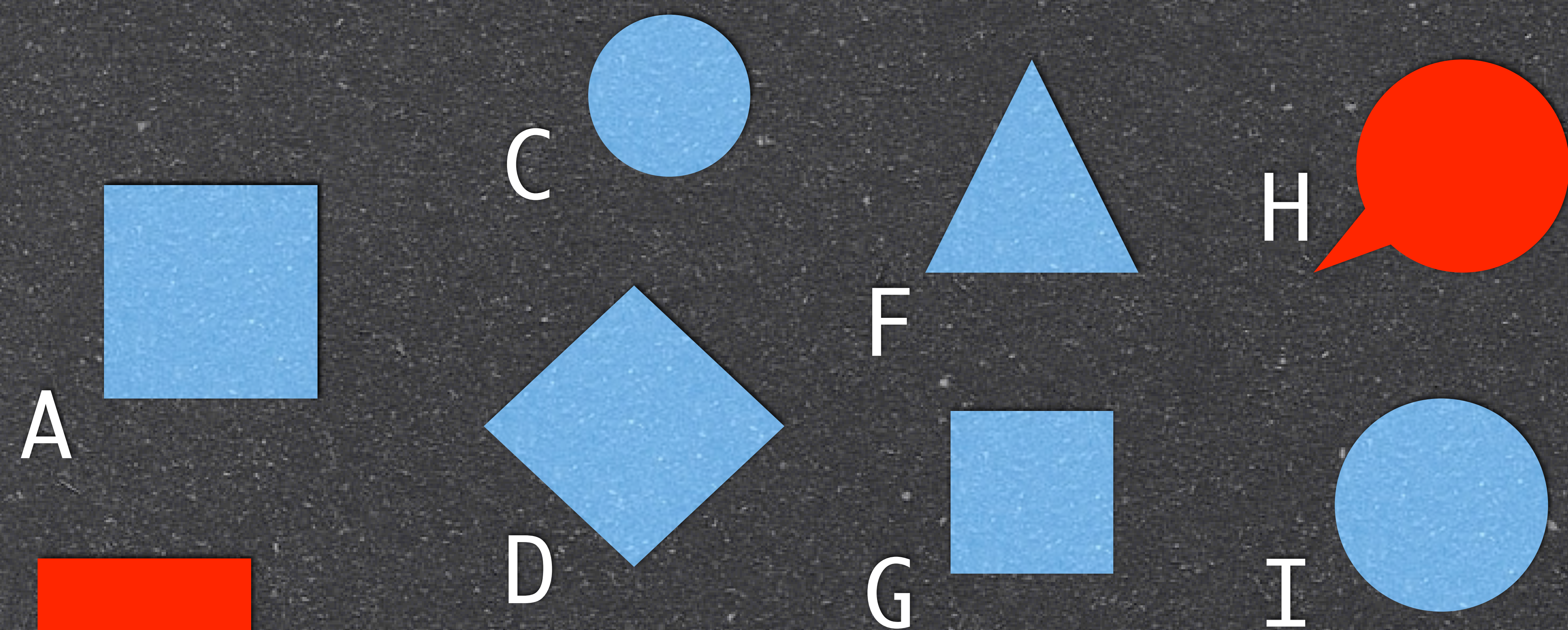
Outline

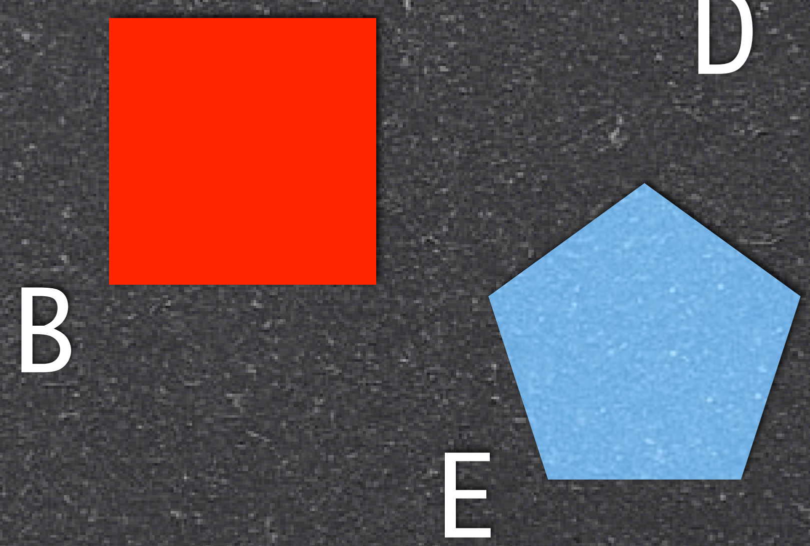
- The meaning(s) of “the same”?
- Congruence, Similarity and other samenesses
- Properties of Dilations
- Dilations for solving Geometry Problems
- Combining Dilations

Which are "the same"? Which one (if any) does not belong?



Which are “the same”?





For which pairs of shapes can you find a rule that makes them “the same” (or not)?

Sameness

- There are many ways in math and in other parts of life to group things as being the same or not.
- Examples outside of math?
- Examples in math? Geometry?

Properties of sameness

- Any A is the same as itself. [For any A, A "same" A.]
- If A is the same as B, then B is the same as A. [A "same" B implies B "same" A.]
- If A is the same as B and B is the same as C, then A is the same as C. [A "same" B and B "same" C implies A "same" C.]

Sameness in Geometry

- In Euclidean Geometry, there are two kinds of sameness that we use most often and study the most: Congruence and Similarity
- Some other samenesses include “equal area” or “equal length”
- A variety of samenesses can be defined by means of transformations

Congruence: Two Versions of the same Definition

- Two figures are congruent if there is a rigid motion that moves one to coincide with the other. (Standard)
- In the plane, two figures are congruent if there is a sequence of translations, rotations, and line reflections that moves one to coincide with the other. (Common Core)

Other possible samenesses in the plane - which are OK?

- Two figures are XXX to each other if there is a translation that moves one to coincide with the other.
- Two figures are YYY if there is a rotation that moves one to coincide with the other.
- Two figures are ZZZ if there is a sequence of translations and rotations that moves one to coincide with the other.

Which figures are XXX, YYY, ZZZ?

Food for thought

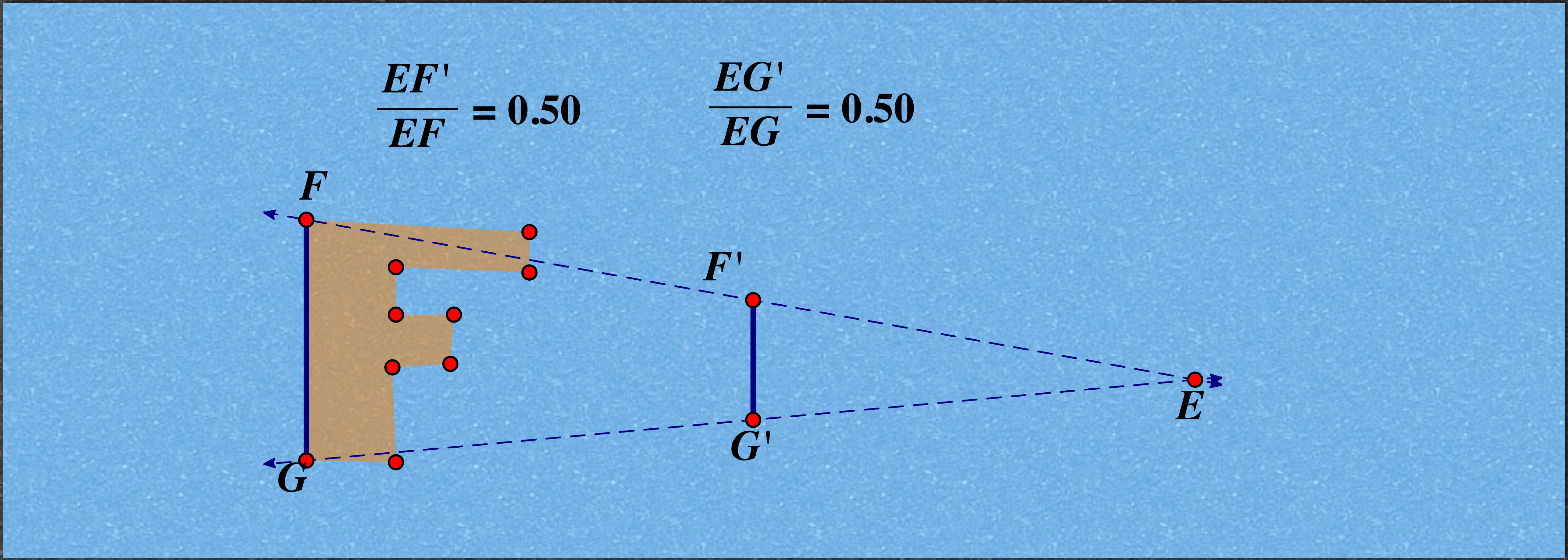
- Which kinds of sameness might relate letters on a normal printed page?
- If a kid thinks a rotated square is a “diamond” and not a square, is she thinking XXX and not just “wrong”.

Similarity

- Two figures are similar if there is a scaling motion that moves one to coincide with the other. (Standard)
- In the plane, two figures are similar if there is a sequence of dilations, translations, rotations, and line reflections that moves one to coincide with the other. (Common Core)

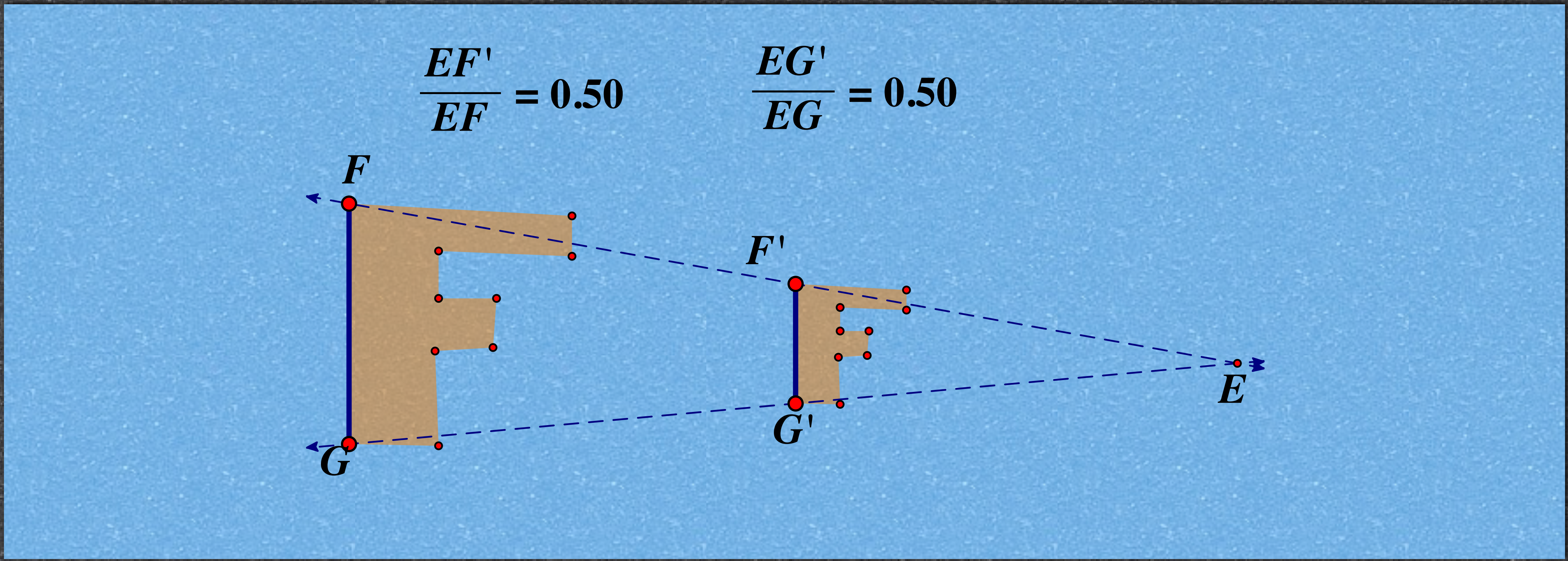
Dilation

- A dilation with ratio r and center E maps a point F to a point F' on line EF so that EF'/EF = r.

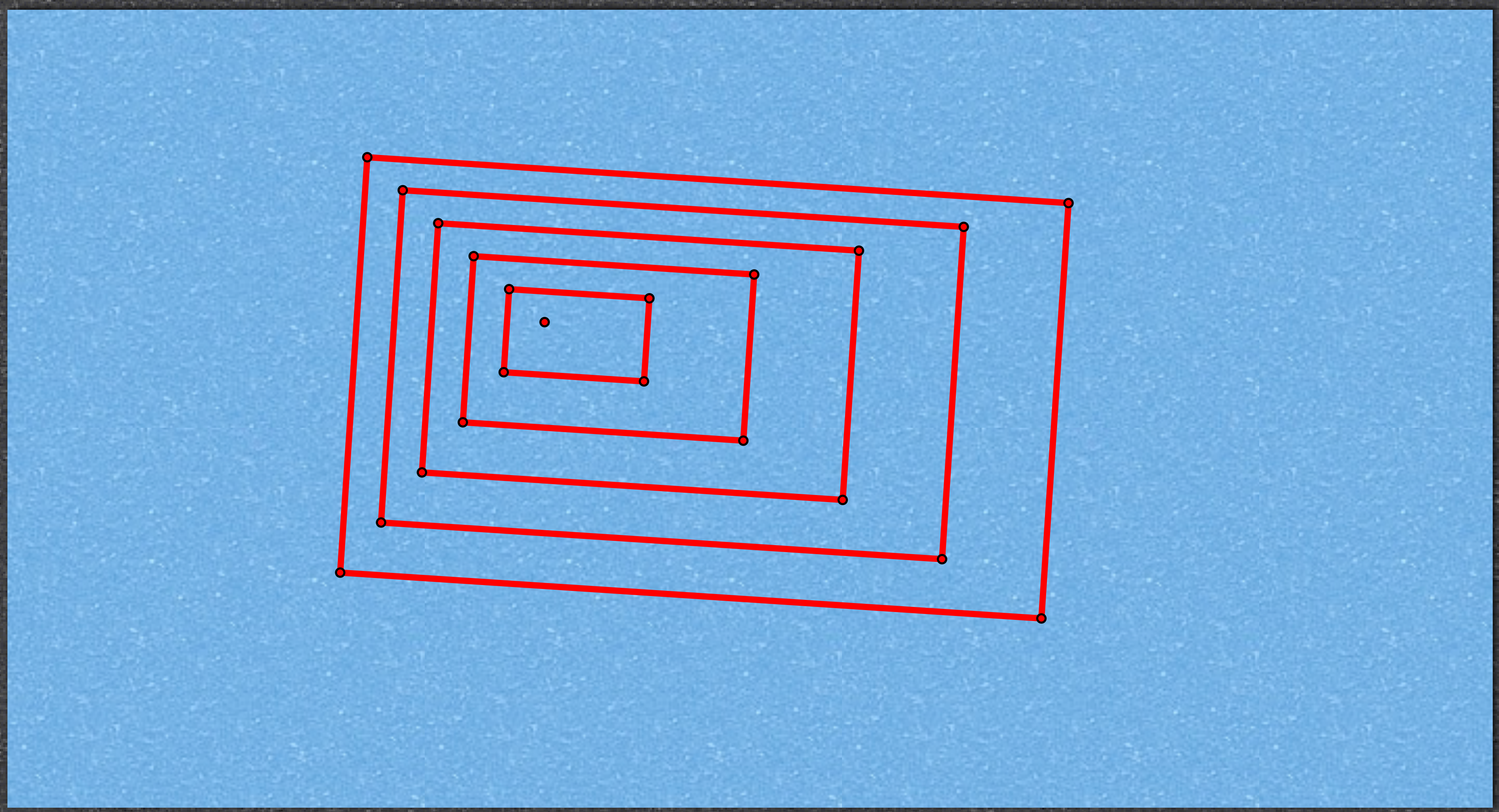


Dilation

- All the other points are transformed by the same rule. All distances are scaled by the same ratio r. Angles are preserved.



Dilations with same center different ratios produce a family of similar figures

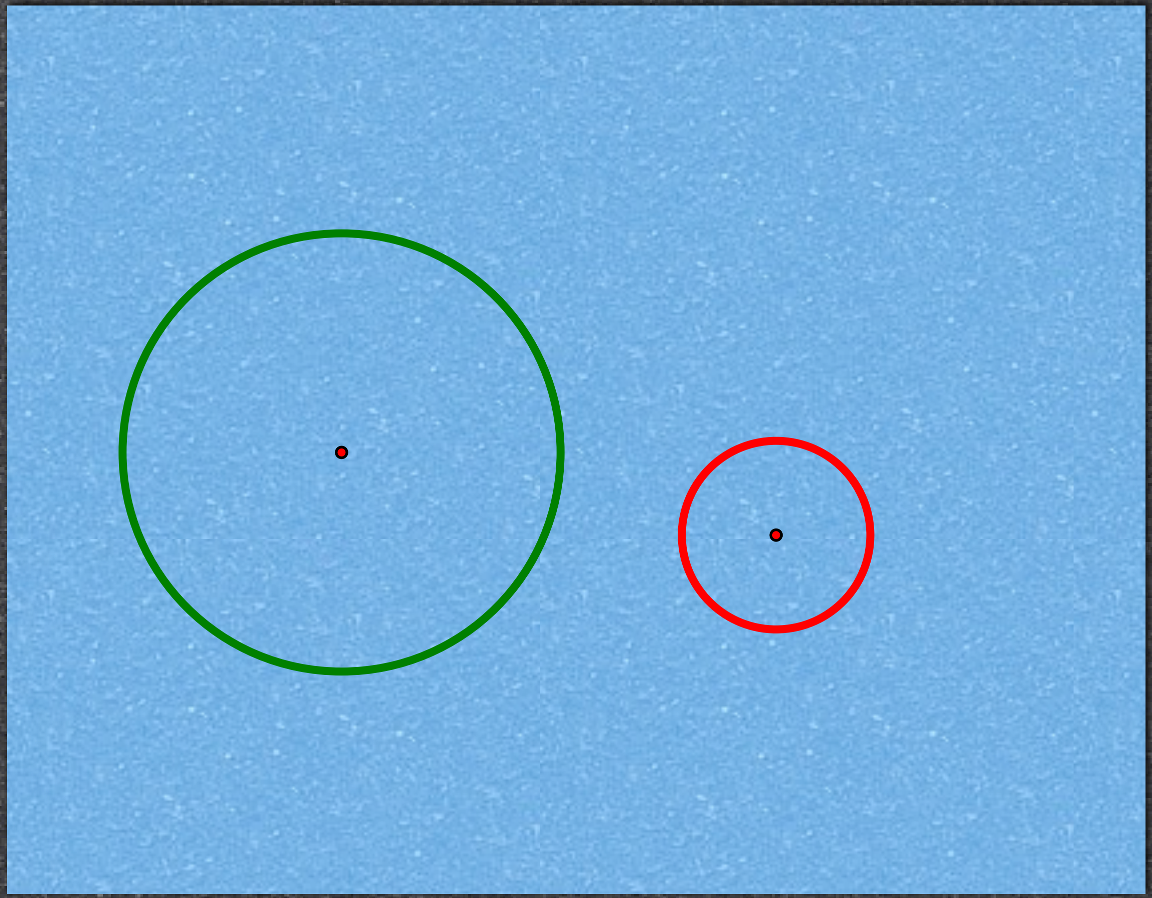


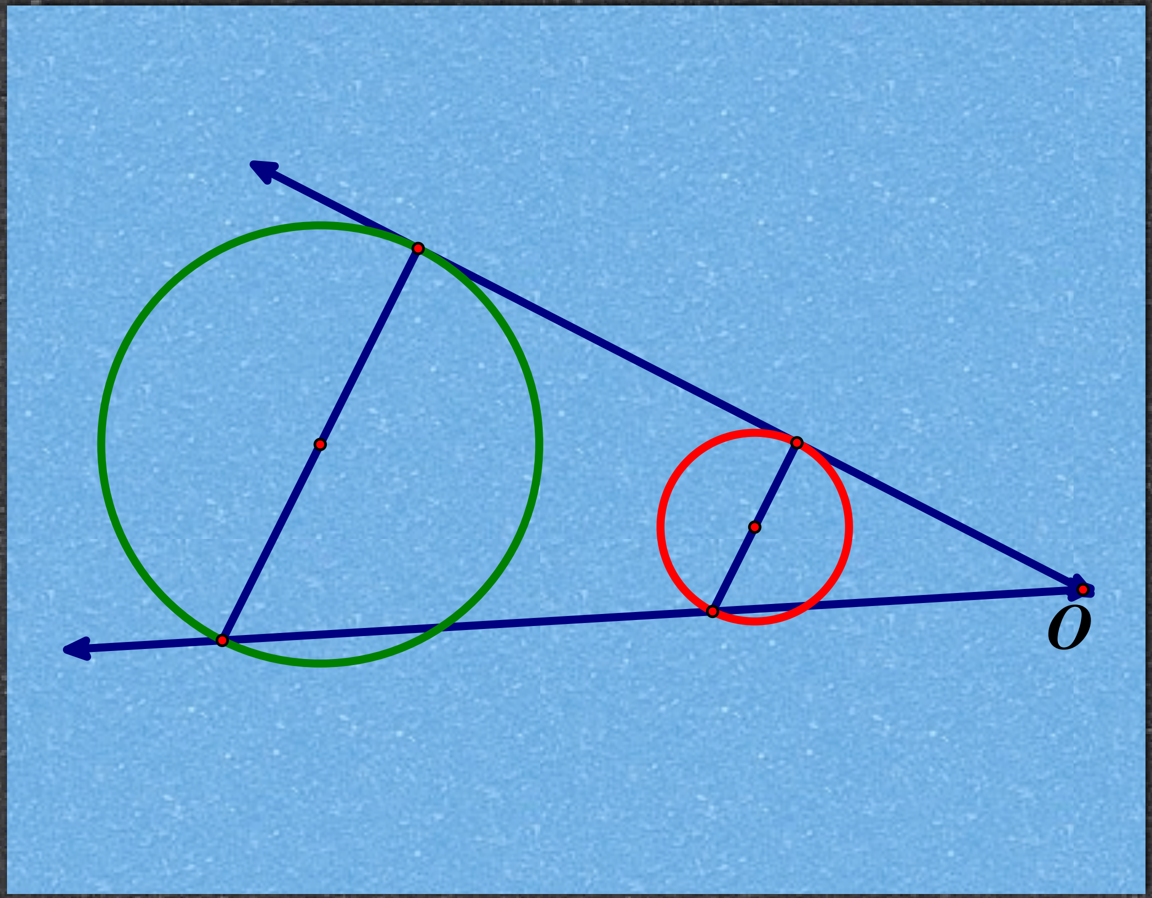
A graph paper example

- Draw a triangle ABC on your graph paper (suggest choosing points with integer coordinates)
- Draw any segment PQ parallel to AB and of different length.
- Draw lines AP and BQ and find the intersection point O. O will be the center of a dilation.
- Draw a line through P parallel to AB and a line through Q parallel to BC. Let R be the intersection of these two lines.
- Now draw the line CR. It should pass through O. And it should be true that OR/OC = OP/OA = OQ/OB

Application 1

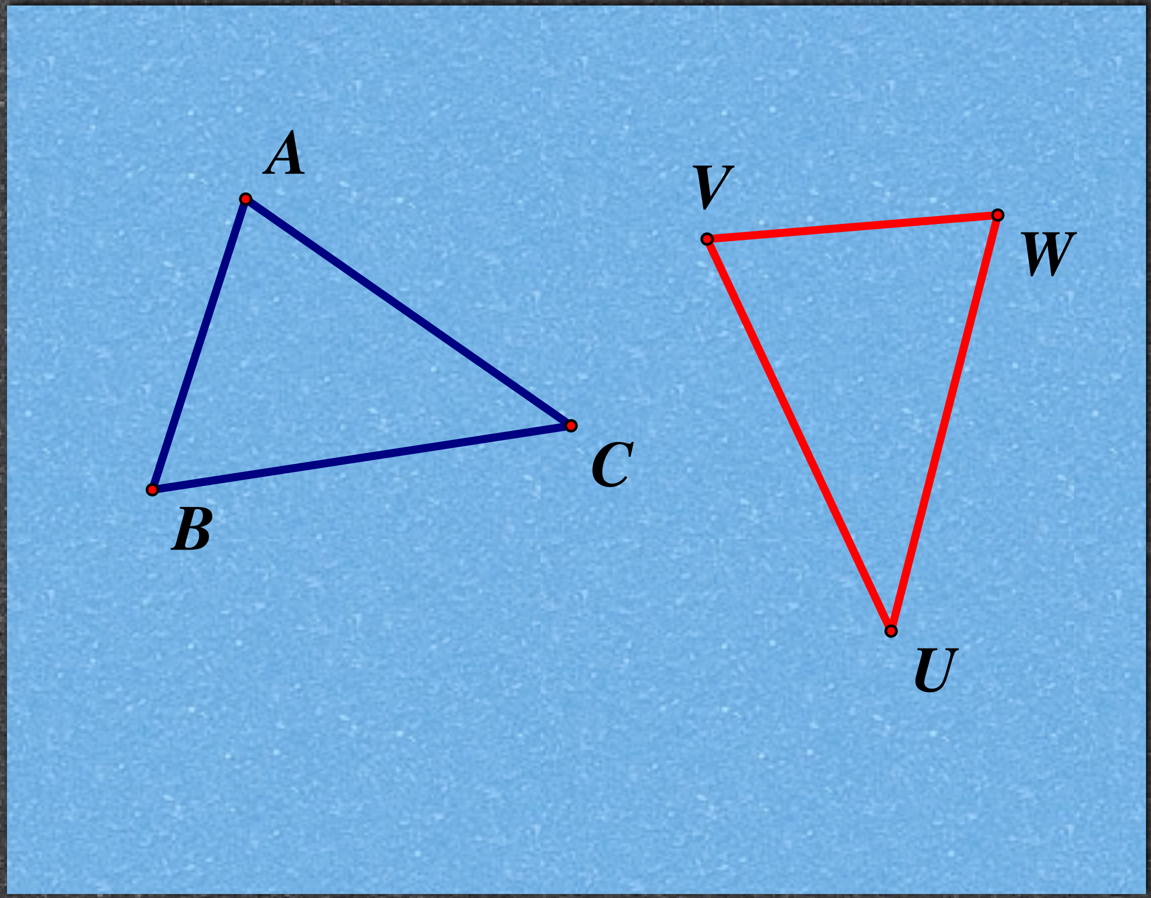
- Given two circles of unequal radius, construct a dilation from one to the other.

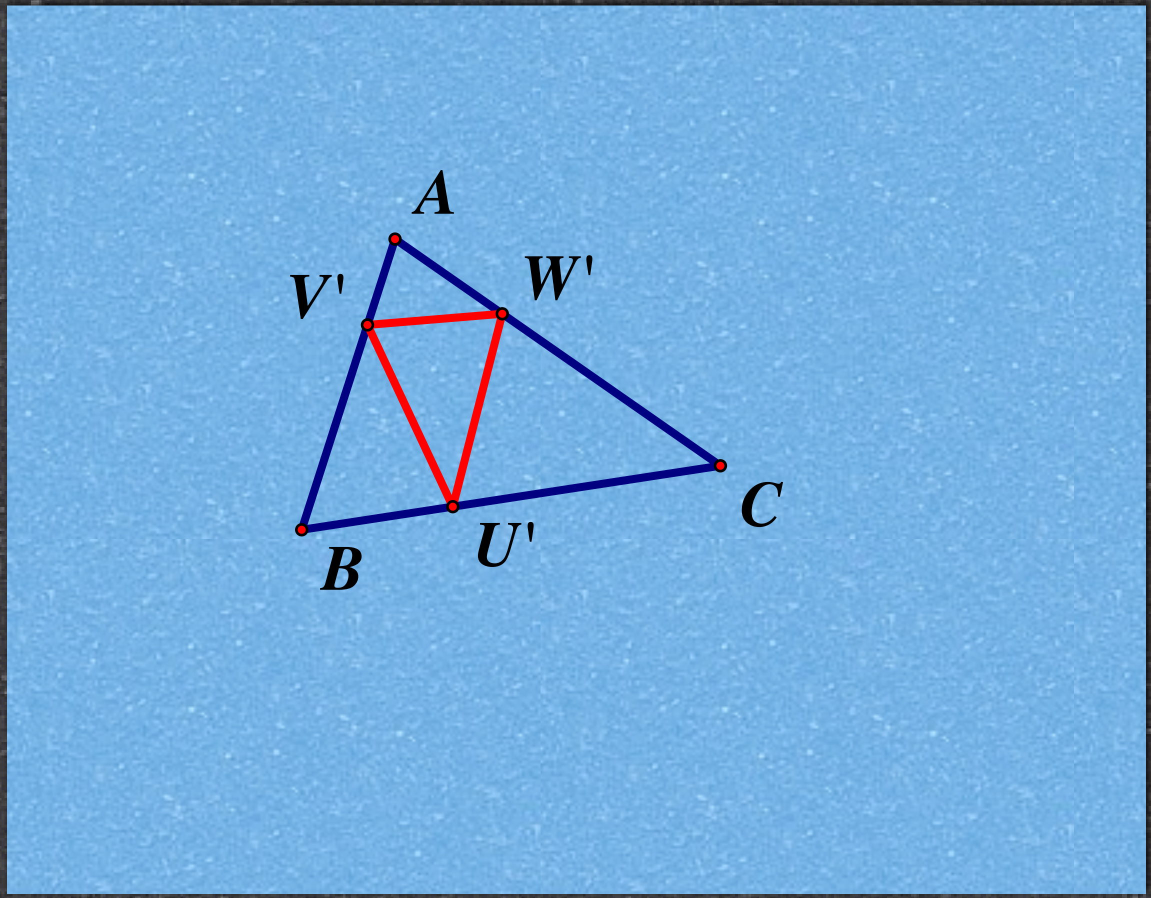




Application 2

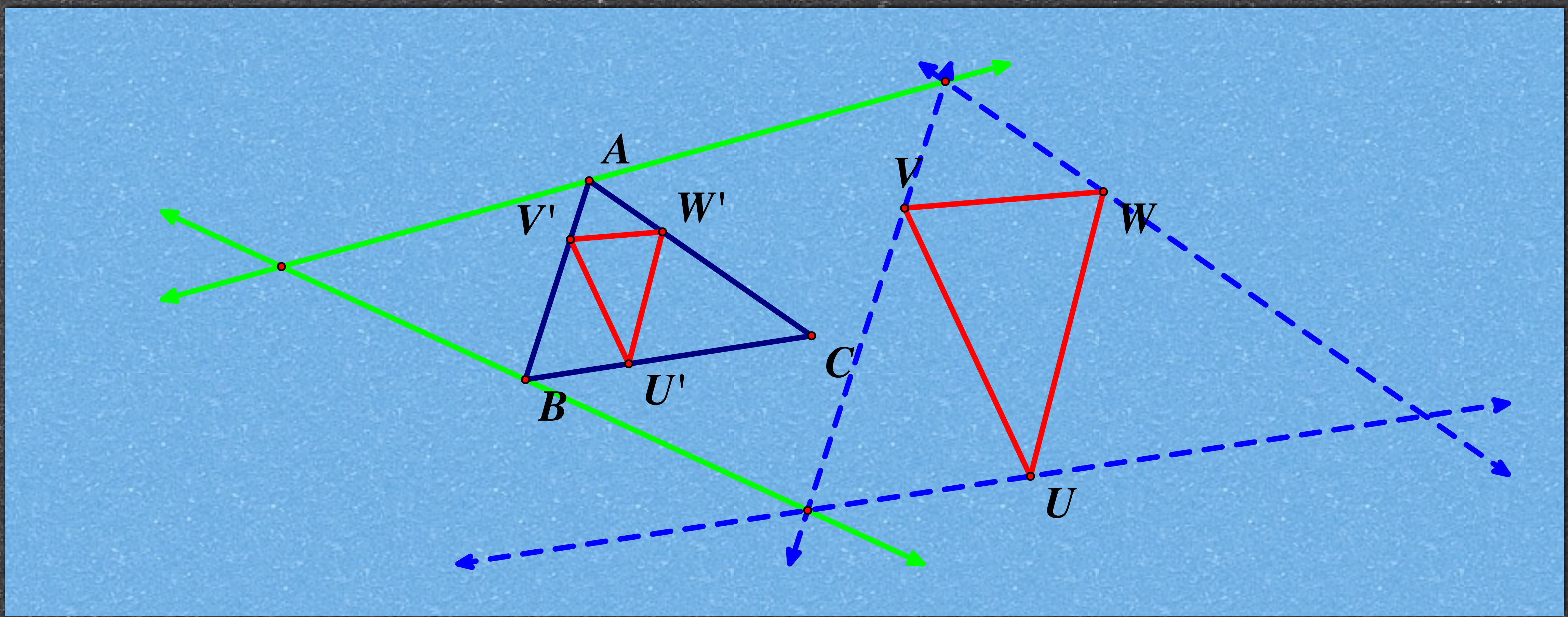
- Given two triangles, ABC and UVW, construct a triangle similar to UVW inscribed in ABC.





Solution 2

- It is easy to construct a triangle similar to ABC circumscribed around UVW by constructing 3 parallel lines. Then dilate..



Composing dilations

- Suppose that a figure is dilated once and then the image is dilated again. Is there a single dilation that will take the first to this second image?
- Experiment: On your graph paper try dilating one segment AB to a second CD and then dilate CD to EF. Can you dilate AB to EF in one step?

Composing dilations

- Suppose that a figure is dilated once and then the image is dilated again. Is there a single dilation that will take the first to this second image?
- Experiment: On your graph paper try dilating one segment AB to a second CD and then dilate CD to EF. Can you dilate AB to EF in one step?
- Answer: Almost true. (What if the first scaling ratio is 2 and the next is 1/2, with difference centers?)

XXX or ???

- Two figures in the plane are homothetic if there is a sequence of dilations that moves the first to coincide with the second.
- Two figures in the plane are homothetic if either there is one dilation or one translation that moves the first to coincide with the second.