A Transformational Perspective on Similarity in Geometry

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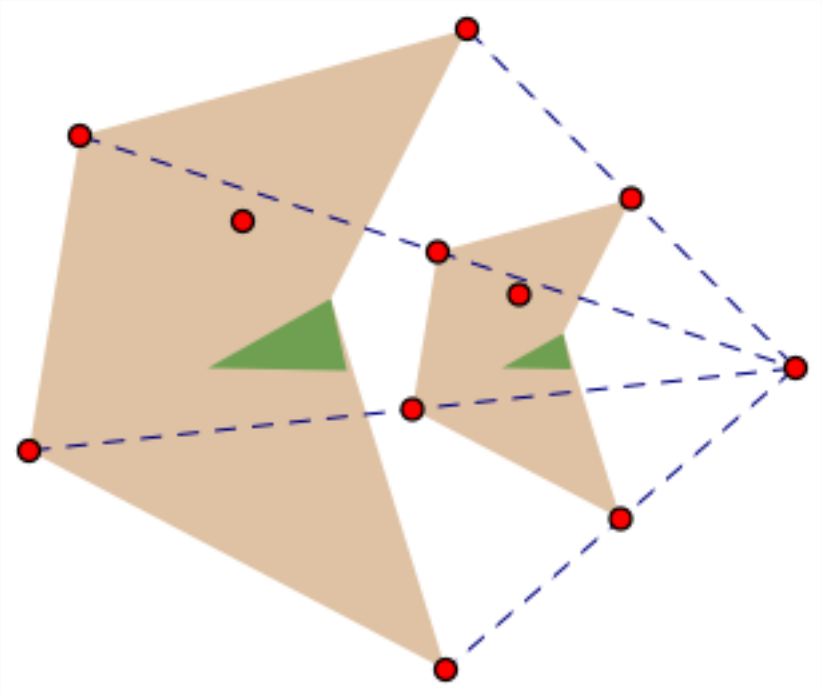
Similarity Transformations

- A Similarity Transformation of the plane (or of space also) is a transformation T that
- Scales distances: there is a scaling constant k > 0 such that |T(A)T(B)| = k|AB| for any points A and B.
- T preserves angle measure
- We often just say "similarity" and leave of the second word.

Definition of Similarity

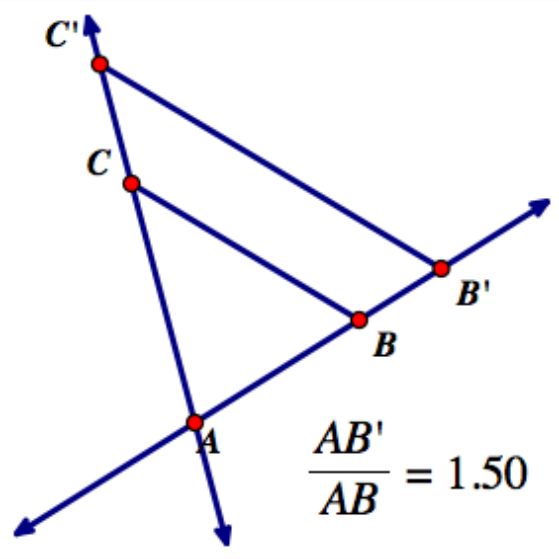
- Two figures F and G are ***similar*** if G = T(F), where the transformation T is a similarity.

Dilations



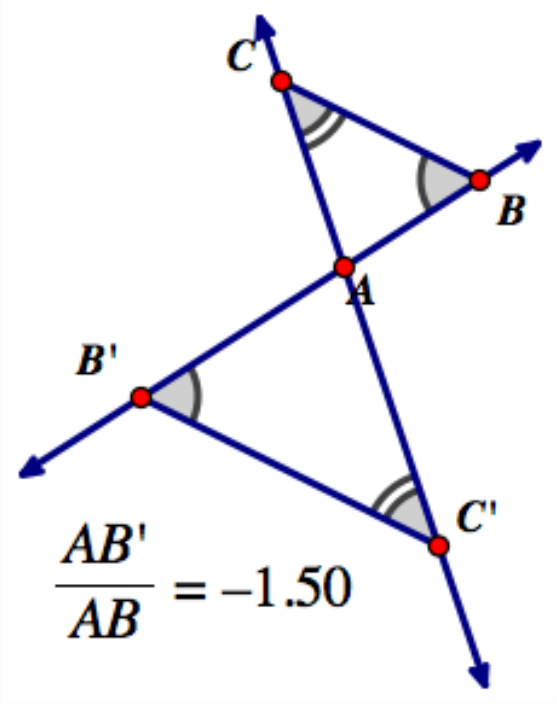
Definition: Dilation

- The **dilation** D with **center** A and **scale factor** k≠0 is a transformation defined thus:
  - D(A) = A
  - If B ≠ A, D(B) is the point on line AB so that AD(B)/AB = k.
  - So the distance |AD(B)| is |k| times distance |AB|, with D(B) on ray AB if k is **positive** and on the opposite ray if k is **negative**.



Dilation Axiom

Let D be a dilation with center A and ratio k. Then D preserves angles and scales all distances by |k|. In other words, for any points B and C, the distance |D(B)D(C)| = |k||BC|.

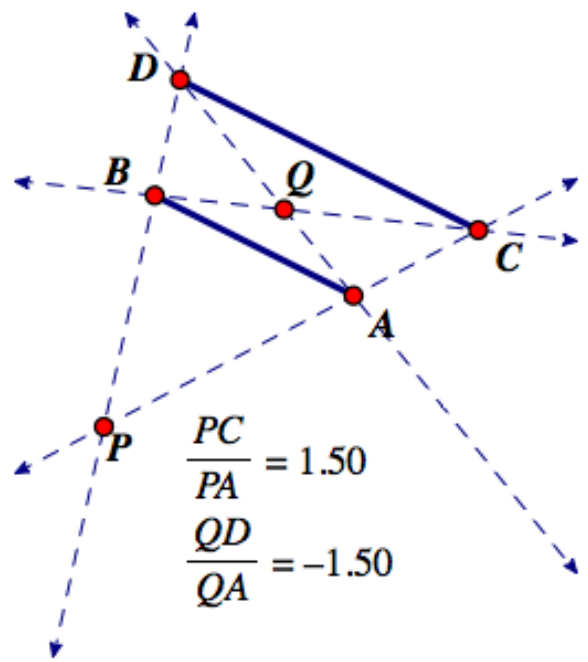


Dilation Images

- Statements to prove about a dilation D with center A:
  1. The D image of a line m not through A is a line parallel to m.
  2. The D image of a line m through A is m.

Dilations and Trapezoids

- Given two parallel segments AB and CD of different lengths, there are two dilations that take AB to CD. One has ratio k = |CD|/|AB| and the other ratio is –k.
- The center P of one is the intersection of lines AC and BD.
- The center Q of the other is the intersection of lines AD and BC.



Breakout 4A

- Your room will have collection of slides, each with a pair of figures. Your goal is to find any centers of dilation from one figure to the other.
- In each slide, drag a blue point to any center of dilation with positive ratio and drag a red point to any center of dilation with a negative ratio.
- Estimate the dilation ratio(s) and type this into the text box.
- No need to rush. Just focus on a good discussion for each case that you do.