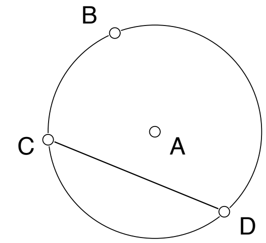
Investigation 2—Perpendicular Bisectors of Chords



Draw a circle with center *A* through point *B* and construct a chord *CD*. You should be able to slide *C* or *D* around without changing the circle.

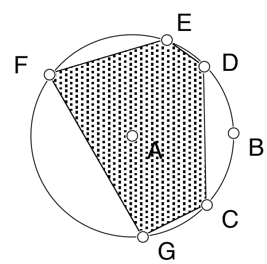
- Construct the perpendicular bisector of *CD* and observe the relation of this line to the circle as you move *C* and *D* around.

☞ What do you see? What is your explanation of what you see?

- Now construct a second chord *DE* and its perpendicular bisector.

☞ How are the two perpendicular bisectors related? How do you explain this?

- Next, draw chord *EC* to form a triangle and construct the perpendicular bisector of this segment. What relationship do you observe and how is this related to earlier explorations?



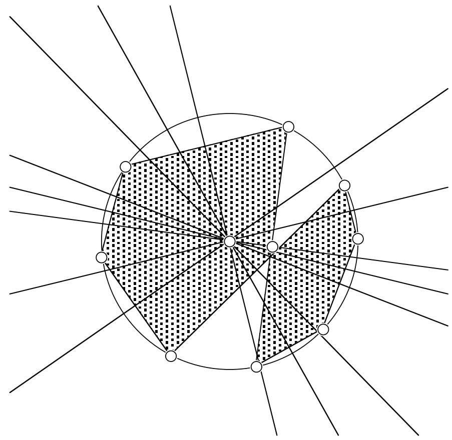
- Hide or delete chord *EC* and its perpendicular bisector and continue creating new chords *EF*, *FG*, and *GC* to form a pentagon. Select points *C*, *D*, *E*, *F*, *G* and choose Polygon Interior from the Construct menu to visualize the polygon better.
- Construct the remaining perpendicular bisectors.
- How are all the perpendicular bisectors related? Why must this be true?

Conclusion

> *A polygon is* **inscribed in a circle** *if all the polygon's vertices lie on that circle.*

Q1. Make a general conjecture about perpendicular bisectors of sides of polygons inscribed in circles.

Q2. Is there a polygon in the figure below? Can you write a conjecture that includes this figure?



Conclusions—Locus Property

- Consider the following two statements about perpendicular bisectors.

> *(1) If* A *and* B *are points and* P *is a point with distance* |PA| *= distance* |PB|*, then* P *is on the perpendicular bisector of* AB*.*
>
> *(2) If* Q *is on the perpendicular bisector of segment* AB*, then distance* |QA| *= distance* |QB|*.*

Q1. Do these statements say the same thing? Explain.

Q2. What reasons would you give to convince someone that these statements are true?

Q3. For each exploration in this section, write whether the exploration is based on, or illustrates, Statement 1, Statement 2, both statements, or neither statement and briefly explain why.

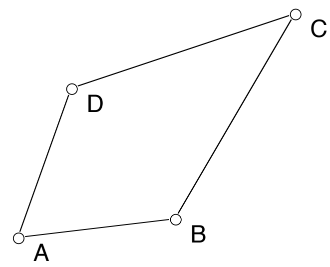
> *Statements (1) and (2) together make up what is called the* **Locus Property of Perpendicular Bisectors***.*

Problems

Here are some problems, questions, or constructions from the work of this section. In each case, answer the question and explain whether you are using statement (1) or (2), both, or neither.

Q1. Let *ABC* be a triangle. If there is a circle that passes through *A*, *B*, and *C*, how do you construct it? When is there such a circle and when is there not?

Q2. Quadrilateral *ABCD* is a *kite* if $|AB| = |AD|$ and $|CB| = |CD|$. How is segment *AC* related to segment *BD*? Explain.

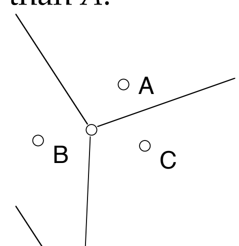


Q3. *Jar Lid Problem: Find the center of a circle.* Draw a circle on a piece of paper by tracing around a jar lid or some other circular object. How can you find the center of this circle? You can create a Sketchpad "jar lid" by drawing a circle, selecting the circle's circumference, copying it, and pasting it into the sketch. Construct the center for such a circle.

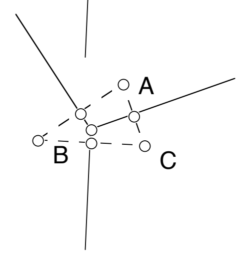
Explore More—Dirichlet Domains

You can think of the perpendicular bisector of segment *AB* as a fence separating the points in the plane closer to *A* than *B* and the ones closer to *B* than *A*.

If you add a third point and want to fence in the three parts of the plane closest to *A*, to *B*, and to *C*, the fences will follow along the perpendicular bisectors but not all along their whole length. In fact the fences will be rays those shown at right.

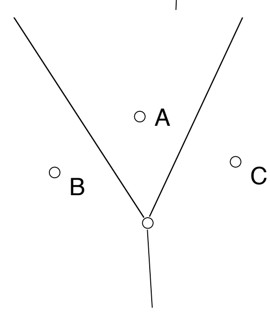


The first figure on the right shows the fences, and the second also shows the triangle *ABC* and its midpoints, as well so that you can see the connection with the perpendicular bisectors and the circumcenter.



The fences can appear a little surprising, even for three points. Can you construct these fences in Sketchpad?

The picture gets even more interesting and complex for four or more points. Given a set of points, if you break up the plane into regions so that each region is the set of points closest to one of the given points, these regions are called *Dirichlet domains*. Dirichlet domains have many applications. (See *Connections*, by Jay Kapraff[§], for more information on Dirichlet domains.)



You can explore the Dirichlet domains of three points, four points, and five points drawn as ripple intersections using demo sketches included with this book.

Challenges

Here are some optional challenge questions, some worthy of projects, which extend ideas from this section.

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§ Kapraff, Jay, *Connections—The Geometric Bridge Between Art and Science*, McGraw-Hill, NY, 1990, ISBN 0-07-034251-2, pp. 220–224.