Exploration 3.2—Ripples

Circles growing from fixed points look like ripples on water. In this exploration you'll study the intersection points of such circles and discover how to construct the circumcircle of a triangle.

Investigation 1—Two Ripples

As two ripples of equal radii grow from two separate points, their intersections produce an interesting trace.

Construction

- Draw segments *AB* and *CD*, as shown below. Label the segment *CD* as *d* (for distance).

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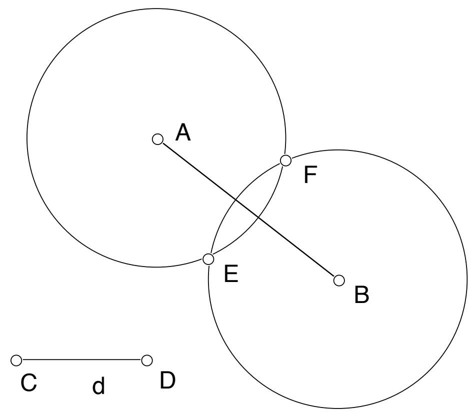
- Construct a circle with center *A* and radius *d*.
- Construct a second circle with center *B* and radius *d*. Drag point *C* or point *D* and observe how the circles move.

Experiment



- Construct and trace points *E* and *F*, the intersection points of the two circles.
- Begin with point *D* close to *C* and then drag *D* to stretch segment *d* to a long length. What geometric object gets traced?
- Starting from points *A* and *B* (and not using points *E* and *F*) construct the object which you conjecture is the trace of points *E* and *F*.
- Construct segments to form polygon *AEBF*. Look for relationships in this polygon. (Measuring may help.)

Conclusions

Q1. What object did you conjecture would be formed by the trace of points *E* and *F*? How did you construct it?

Q2. Explain why you think that your object really corresponds to the trace.

Q3. What kind of polygon do you conjecture *AEBF* is? What reasons can you give for your conjecture? What special relationships did you find?

Q4. In the figure, the circles *AE* and *BE* both pass through points *E* and *F*.

Construct the circles *FA* and *EA*. These circles pass through point *B* as well as through point *A*. Explain why.

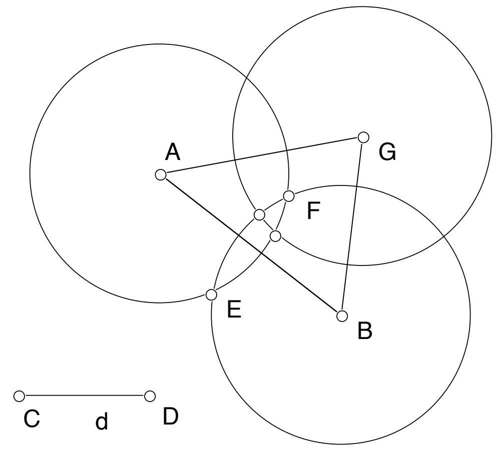
Show how you can apply the results of Exploration 3.1 to the pair of circles *AE* and *BE* and the pair of circles *FA* and *EA*. to explain the relationship between segment *EF* and segment *AB*.

Investigation 2—Three Ripples

This investigation continues by adding a third ripple to the figure.

Construction

- Add a third point and a third ripple. All three circles should have a radius equal to the length of segment *CD*.
- Then construct the intersection points of the new circle with the original two circles. Trace these four new points.



Experiment

- Drag point *D* and observe the trace of the six points. Describe what you see.
- There is one choice of length *CD* for which the three circles appear to pass through the same point. Measure distances *FA*, *FB*, and *FG*. What do you observe about these distances when the three circles appear to intersect at a common point?

Conclusions

Q1. Must there always be a radius such that the three ripples come together at a common point? What happens to the intersection points when this happens?

Q2. Explain why it must be that, if the three ripples intersect at a common point, the distances *FA*, *FB*, and *FG* are equal.

Explore More—Four Ripples

Investigate what happens when you add a fourth ripple to the three-ripple sketch. Trace all the intersection points of ripples. Use colors to distinguish traces if you have a color monitor.

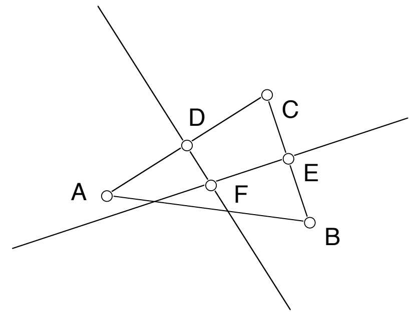
How many intersection points will you be tracing? How many lines will appear in the traces and how are they related? Can you construct them? If you connect the four centers by segments to form a quadrilateral, how are these segments related to the ripple traces?

Exploration 3.3—Perpendicular Bisectors in a Triangle

Investigation 1—Intersection of Two Perpendicular Bisectors

In this investigation, you will construct two perpendicular bisectors of a triangle and examine a trace of their intersection.

Construction



- As shown at right, draw a triangle *ABC* and construct the perpendicular bisectors of sides *AC* and *BC*.
- Let point *F* be the intersection of the two perpendicular bisectors. Trace it.

☞ Observe the trace of point *F* as you drag point *C*. What do you conjecture the trace of *F* is?

☞ Measure the distance from *F* to other points. What relationships do you find among the distances?

- Construct the object which you conjecture is the trace of point *F*. (Do the construction using points *A*, *B*, and *C*, without using point *F*.) Check that *F* travels along this object as you move point *C*.
- Construct the circle with center *F* through point *C*.

☞ What do you observe? What lines in the figure are diameters of the circle?

- Trace the circle as point *C* is dragged.

☞ How is this exploration related to the earlier investigations in this section?

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- Make the circle dashed. Now select the three points *A*, *B* and *C* and choose Arc Through 3 Points in the Construct menu. Make the arc solid.

☞ Observe how the arc is related to the circle. Notice how the center of the arc can be constructed.

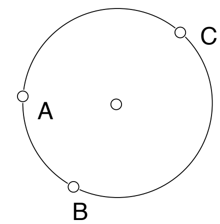
Conclusions

> *The* **circumcircle** *of a triangle is the circle passing through all three vertices of the triangle. The center of this circle is called the* **circumcenter** *of the triangle.*

Q1. State your conjecture about the shape of the trace of point *F* and explain why you think it is true.

Q2. The circle *FC* appears to be the circumcircle of triangle *ABC*. Write your explanation for why this is true.

Construction—Circumcircle Script



- In your sketch of the circumcircle, hide your construction lines and extra objects so that the figure looks like the one at right.
- Select all the objects. The press on Custom Tool Button, the double-arrow symbol at the bottom of the Toolbar on the left and choose Create New Tool … Name your tool with a descriptive name (e.g. circumcircle of ABC).
- Now your tool appears in the Custom Tool Button menu. If you select this tool, you can click on three points (old or new) to construct the circle through the 3 points.

Explore More— Quadrilateral Challenge

Now try some of the same ideas with a quadrilateral *ABCD*. Construct the perpendicular bisectors of the sides and construct the circle through *ABC*. What relationships do you see when point *D* is dragged so that this circle passes through it? How are the perpendicular bisectors of the sides related in general? Why don't they behave like the perpendicular bisectors of a triangle?

Custom Tool Button