Exploration 5.2—Coins, Distance to Lines, and Tangent Circles

There is more to discover from circles tangent to two lines. Draw two intersecting lines on paper and slide identical coins to positions where they are tangent to both lines. How many coins can you place this way? What is the arrangement of coins?

Investigation 1—Visualizing Circles Tangent to Two Lines

Before beginning this investigation with Sketchpad, try out the version on paper with the coins described above and then make some conjectures.

Construction—A Stack of Coins

You need a stack of coins which you can make constructing circles of the same radius.

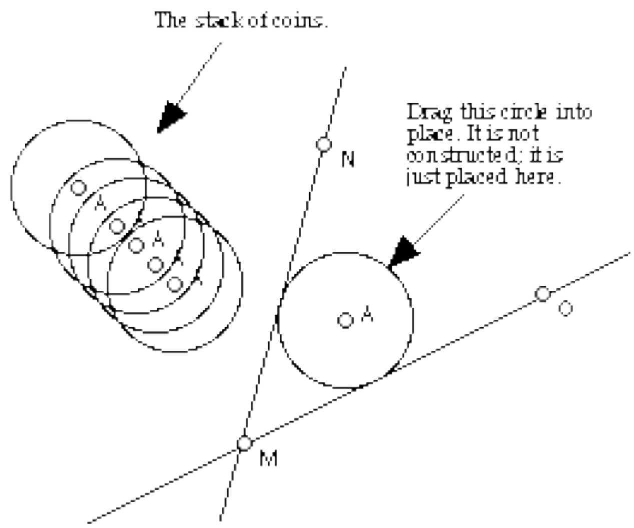
- In a new sketch, draw a segment r and a half-dozen points A, B, C, D, E, F. Select all the 6 points and also the segment. Construct six circles with center A, … , F and the same radius r.
- Draw two intersecting lines in the sketch.

Experiment

- Take one of the circles and drag it around the screen looking for a place where the circle seems approximately tangent to both lines. When you find such a position, leave the circle there and move another duplicate circle to another position tangent to both lines. Continue until you have found all the positions where you can put the circle tangent to both lines.

☞ Observe how many tangent circles there are and how they are arranged.

- Connect the centers of these tangent circles with segments to form a polygon. What is the name of a polygon of this shape? (Remember that since the circle locations were not constructed, this figure will only be approximate.)



- Drag the intersecting lines until they are (approximately) perpendicular and move the circles so they are tangent again.

☞ Observe what shape is formed by the centers of the circles in this case.

Conclusions

Q1. In the experiment with two intersecting lines, how many circles could be placed tangent to both lines?

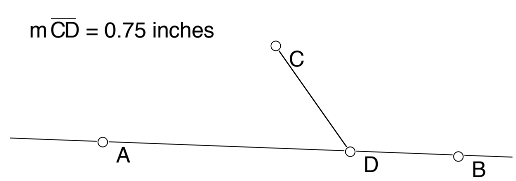
Q2. How were the tangent positions for the coins related? What geometric object was formed by the centers of the tangent circles? Explain why.

Investigation 2—The Distance from a Point to a Line

You know from experience how to pick the shortest path across the street, but consider the geometry of the situation.

Construction—Distance Slider

- Start with a line *AB* and a point *C*. Construct a point *D* on the line so that *D* slides along the line.
- Construct the segment *CD* and measure its length.



Experiment

- Slide point *D* along line *AB* and find the position of *D* at which length *CD* is smallest.

☞ Observe how the segment and the line are related when the length of segment *CD* is minimal.

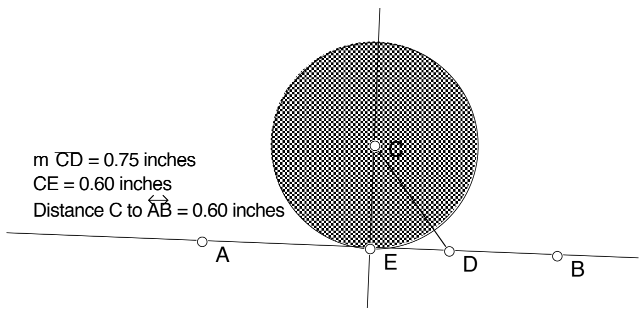
- Make a conjecture about the angle between the segment and the line when the length of *CD* is minimal. Make measurements to support your conjecture.

Conclusions

Q1. Write down your conjecture about the angle between segment *CD* and line *AB* when the length of segment *CD* is minimal.

Construction—Other Measurements

- Construct the line through point *C* perpendicular to line *AB* and construct point *E* as the intersection of these lines.
- Measure distance *CE*. Compare with distance CD.
- Construct the circle with center *C* through *E*. Also construct the circle's interior and shade or color it lightly.
- Select point *C* and line *AB* and choose Distance from the Measure menu to measure the *distance from point* C *to the line.* What other measure does this equal?



Experiment

- Construct a point F on the line so that AD = AF. Slide point *D* along the line again and check that your AD still equals AF.
- How is the circle with center A through D related to F?

Conclusions

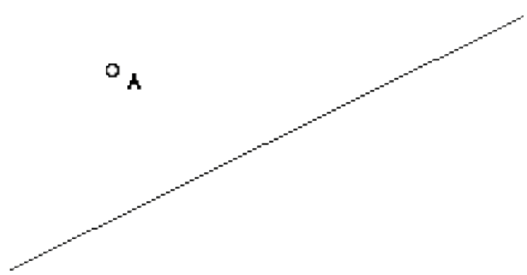
Q1. Which points P on the line have distance AP > AE?

Q2. Which points on the line have distance AP < AD?

Q3. What kind of triangle is *CDE*? Use this fact to find an inequality explaining the relationship between distance *CD* and distance *DE.*

Q4. The figure at right shows a line and a point. Measure the distance from the point to the line in inches and explain your method.

Q5. Sketchpad's Measure menu contains a command for measuring the *distance from a point to a line*. Write an explanation of what this concept means and how this distance can be computed with Sketchpad by measuring a distance between two points.



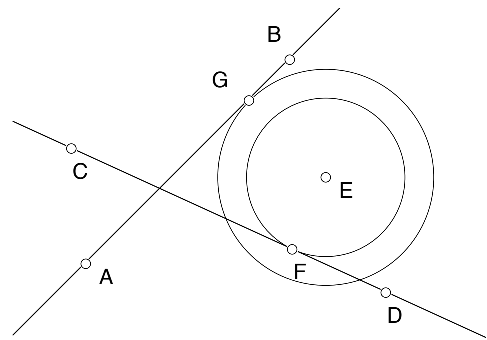
Investigation 3—Coincident Tangent Circles

Your goal is still to construct circles tangent to two lines. In this investigation, you will consider the question, "When is a point the center of a circle tangent to two lines?"

You will construct a partial solution to the question by starting with a center point and constructing a circle tangent to each of two lines and then drag to find under what conditions the circles coincide.

Construction—Two Lines, Two Tangent Circles

- In a new sketch draw two intersecting lines *AB* and *CD*.



- Construct two circles each with center at free point *E*, one tangent to line *AB* and the other tangent to line *CD*. You may wish to color the two circles different colors.
- Label the points of tangency *F* and *G* as in the figure at right. Construct point *H* as the intersection of the two lines.
- Construct radius segments *EF* and *EG* and also segment *EH*.
- Measure the lengths of segments *EF* and *EG* and the distance from point *E* to each of the lines. Also measure distance *EH* and angles *EHF* and *EHG*.

Experiment

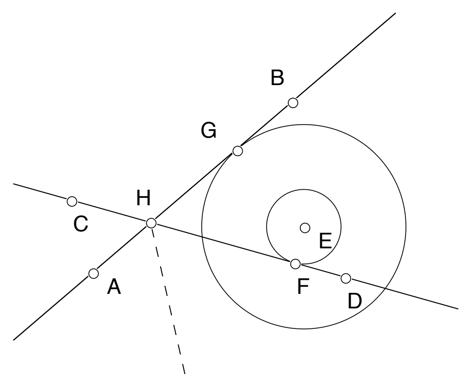
If a point *E* is the center of a circle tangent to a line, you know that the radius of the circle is equal to the distance from *E* to the line.

The two circles constructed above are different for most positions of point *E* but sometimes the two circles will coincide, and the distances to the lines will be equal.

- Drag point *E* around and look for positions where the circles coincide. *Be sure to move* E *around to test all parts of the plane*, not just the region inside one angle.

☞ Compare the triangles *EHF* and *EHG* when the circles coincide. You may want to measure some distances and some angles.
  - ✓ Look for distance measurements that are equal when the circles coincide.
  - ✓ Look for angle measurements that are equal when the circles coincide.
  - ✓ Look for triangles that are congruent when the circles coincide.

Construction—Angle Bisectors and Tangent Circles



- Continue with the same sketch. Construct the angle bisector of angle *AHD*. Make this ray dashed using the Line Weight command in the Display menu.

Experiment

- Drag point *E* to a location on the bisector.

  Observe what happens to the circles and measurements when point *E* is located on the bisector. Look for reasons to explain what you see.
- The line through point *H* perpendicular to the bisecting ray looks like it might prove useful. Construct it and make it dashed.
- The bisecting ray needs continuation on the other side of point *H*. Find a way to accomplish this and make it dashed.

☞ Observe what happens to the circles if *E* is dragged to a location on any of the dashed lines. Look for reasons to explain what you see.

Conclusions

Q1. What kind of triangles are *EHF* and *EHG*? When the circles coincide, how are these triangles related? Write down a reason that explains why this is true.

Q2. Complete the following conjecture:
"If a point *E* is the center of a circle tangent to both line *AB* and line *CD*, *E* is located_________."

Q3. What happens to the circles if point *E* is located on ray *n*, the bisector of angle *AHC*. What is the reason for this?

Q4. Explain what happens to the circles if point *E* is located on line *o*, the perpendicular through *H* to ray *n*. What is the reason for this? How is this reason related to your answer to Q3?

Q5. Use your conjecture about the location of the centers to construct (using either Sketchpad or paper) several circles tangent to intersecting lines *AB* and *CD*.