4.3

Fig 1

Given: ABCD is quadrilateral

MNOP is a quadrilateral formed by connecting the midpoints of the sides of ABCD, so AM = MB, BN = NC, BO = OC, and CP = PD.

Prove: Quadrilateral MNOP is a parallelogram.

1. Draw diagonal DB of ABCD.
2. From the given, we can say that AD = 2AP and that AB = 2AM.
3. Since <A is shared between ΔDBA and ΔPMA, then by SAS for similarity, these two triangles are similar with a scale factor of 2.
4. Therefore, DB = 2PM.
5. Similarly, by given CD = 2CO and CB = 2CN and since <C is a shared angle, then ΔDBC is similar to ΔONC with a scale factor of 2.
6. Therefore, DB = 2ON.
7. From 4 and 6, 2PM = 2ON, so PM = ON. Therefore, two opposite sides of the midpoint quadrilateral are equal.
8. Furthermore, from given DA = 2DP and DC = 2DO and since <D is a shared angle, then ΔDAC is similar to ΔDPO with a scale factor of 2.
9. Therefore, AC = 2PO.
10. From given BA = 2BM and BC = 2BN and since <B is a shared angle, then ΔBAC is similar to ΔBMN with a scale factor of 2.
11. Therefore, AC = 2MN
12. From 9 and 11, 2PO = 2MN, so PO = MN. Therefore, two opposite sides of the quadrilateral are equal.
13. From 7 and 12, we can conclude that both sets of opposite sides are equal.
14. From properties of a parallelogram; a quadrilateral is a parallelogram if and only if the opposite sides are equal.
15. Therefore, we can conclude that MNOP is a parallelogram.

4.4

Fig 2

Given: ABCD is a kite.

MNOP is a quadrilateral formed by connecting the midpoints of the sides of ABCD, so AM = MB, BN = NC, BO = OC, and CP = PD.

Prove: MNOP is a rectangle.