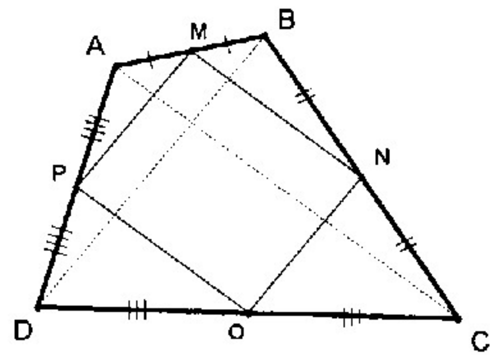
4.3

Given: Quadrilateral ABCD with midpoints M of AB, N of BC, O of CD and P of AD
To Prove: MNOP is a parallelogram



Proof:

- Draw in diagonals AC and BD and segments MN, NO, OP and PM
- MB = ½ AB, NB = ½ CB and OD = ½ CD, PD = ½ AD by definition of midpoint
- $\angle$ MBN = $\angle$ ABC and $\angle$ ODP = $\angle$ CDA (each the same angles)
- ΔABC ~ ΔMBN and ΔODP ~ ΔCDA by SAS
- $\angle$ ACB = $\angle$ MNB and $\angle$ OPD = $\angle$ CAD by definition of similar
- AC || MN and OP || AC by Theorem 14
- Therefore MN || OP by Corollary 13.b
- CO = ½ DC, CN = ½ BC and AP = ½ AD, AM = ½ AB by definition of midpoint
- $\angle$ DAB = $\angle$ PAM and $\angle$ DCB = $\angle$ OCN (each the same angles)
- ΔDBA ~ ΔPMA and ΔDCB ~ ΔOCN by SAS
- $\angle$ DBA = $\angle$ PMA and $\angle$ CDB = $\angle$ CON by definition of similar
- PM || DB and DB || ON by Theorem 14
- Therefore PM || ON by Corollary 13.b
- Since MN || OP and PM || ON, MNOP is a parallelogram