Homework 1 for 509A, Homological algebra
due Wednesday, April 17, 2019

Throughout this homework, R will be a associative ring with 1.

Problem 1. Show that the category of chain complexes over R is an abelian category.

Problem 2. Show that a chain map $f : C_\bullet \to D_\bullet$ induces a well-defined map on homology: $H_n(f) : H_n(C_\bullet) \to H_n(D_\bullet)$.

Problem 3. Exercise 1.4.5 in [1]. 1.4.5.4 is optional but highly recommended to ponder over.

Problem 4. Show that a chain complex $C_\bullet$ is split exact if and only if it is null homotopic.

Problem 5. Show that a chain complex $P_\bullet$ is a projective object in the category of chain complexes over a ring R if and only if $P_\bullet$ is a split exact complex of projective modules.

REFERENCES

[1] C. Weibel, An introduction to homological algebra, Cambridge University Press, 1995