

PRE-MIDTERM TOPICS, MATH 504, FALL 2018

General principle: the expectation is that you are well versed in everything covered in lectures and homework. In particular, know at least one solution to all and any homework problems, and maybe several for some of them. The list below is not claimed to be comprehensive but I tried to mention most of the topics we covered. If you notice an omission, let me know!

- (1) Basic concepts:
 - (a) Groups, subgroups, homomorphisms, cosets and double cosets, normal subgroups, factor groups
 - (b) Group actions, stabilizers, centralizers, normalizers
 - (c) Presentations by generators and relations
 - (d) Exact sequences, split exact sequences for groups
 - (e) p-groups, Sylow subgroups
 - (f) Direct and semi-direct products
 - (g) Center, commutator subgroup
 - (h) Filtrations, derived series, central series
 - (i) Solvable and nilpotent groups (several equivalent descriptions)
 - (j) Free groups, free products, amalgamated free products
 - (k) Categories and functors.
- (2) Fundamental examples:
 - (a) Symmetric groups (everything about them you learned from the worksheet),
 - (b) dihedral groups (various presentations)
 - (c) cyclic and abelian groups
 - (d) groups of small order
 - (e) matrix groups.
- (3) Theorems:
 - (a) Cayley
 - (b) Lagrange
 - (c) Three isomorphism theorems
 - (d) Class formula
 - (e) Jordan canonical form
 - (f) Structure theorem for finitely generated abelian groups
 - (g) Sylow theorems (two proofs for the first theorem)
 - (h) Jordan-Holder theorem; Zassenhaus lemma