Wallpaper Group

December 8, 2021

Symmetry

Symmetry: is a transformation that leaves an object unchanged.

- Nature
- Art
- Science

Group

- associativity
- identity
- inverse

Dihedral Groups

A group of symmetry of a regular polygon that consist of rotation and reflections.

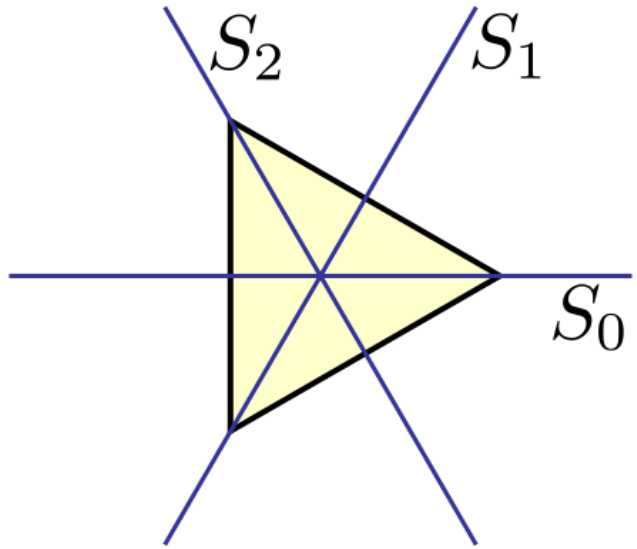


Figure: Picture of D_3 with its lines of reflection

Isomorphism

Isomorphic: if there is a bijection ϕ from G to G' which satisfy $\phi(xy) = \phi(x)\phi(y)$

- Bijection tells us G and G' are the same size
- $\phi(xy) = \phi(x)\phi(y)$ tells us that G and G' are the same.

Isomorphism

- Mulitplication modulo 8

	1	3	5	7
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

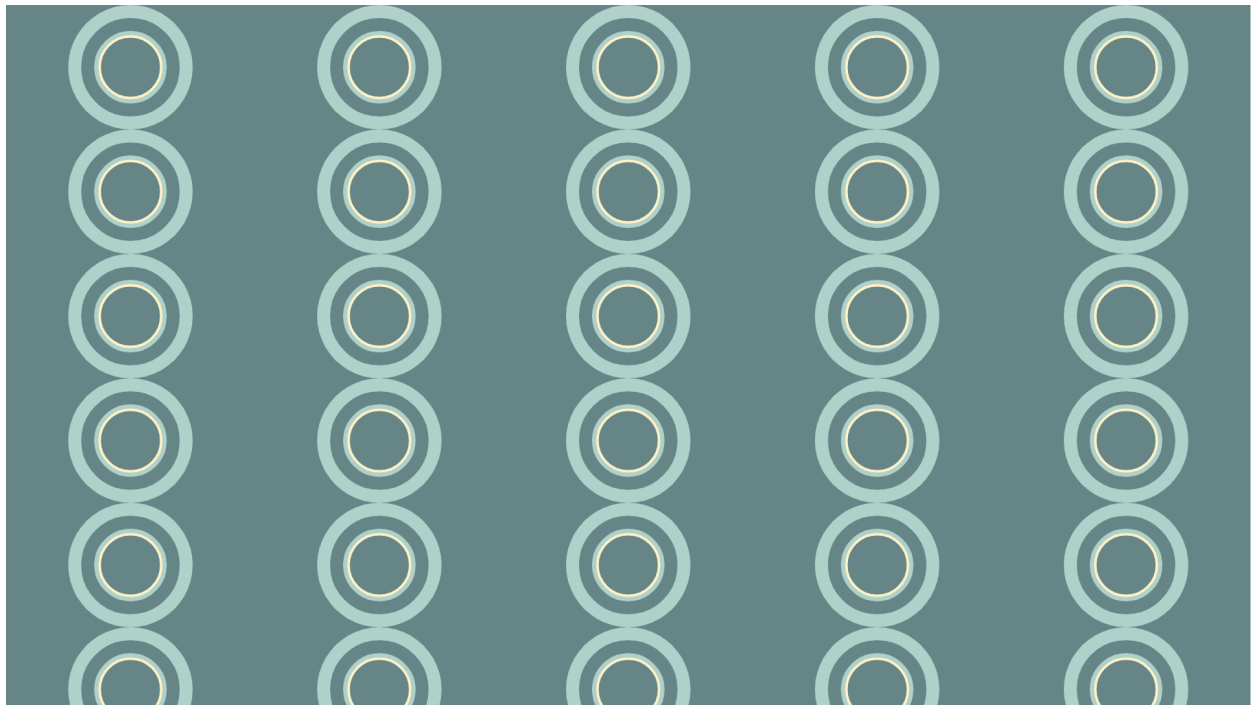
Symmetry in 1D

In 1D were limited were limited to transformation that create symmetry.

- Identity
- Translation
- Reflection

Symmetry in 2D

- Identity
- Reflection
- Translation
- Rotation



Point Group

Point Group: A fixed point on the plane that we rotate/reflect around.

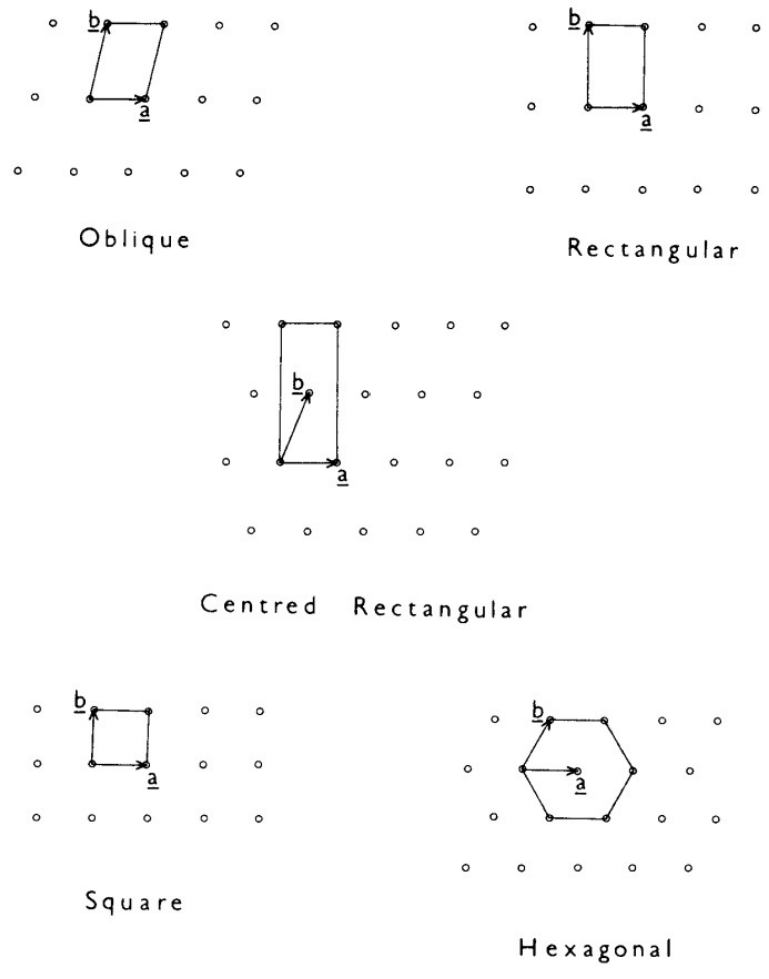
- A subgroup of the wallpaper group

$$\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \qquad \begin{bmatrix} cos2\theta & -sin2\theta \\ sin2\theta & cos2\theta \end{bmatrix}$$

Lattices

Lattices: Think of it as translation.

- A subgroup of the wallpaper group
- 5 types of Lattices



Wallpaper Group

There are 17 different types of wallpaper group.

- There does not exist an isomorphism between any of the of the wallpaper group.
- p1: oblique lattice and it only has the identity as it's symmetry.
- P4mm: square lattice and consist of 4 rotation and a reflection.

P1

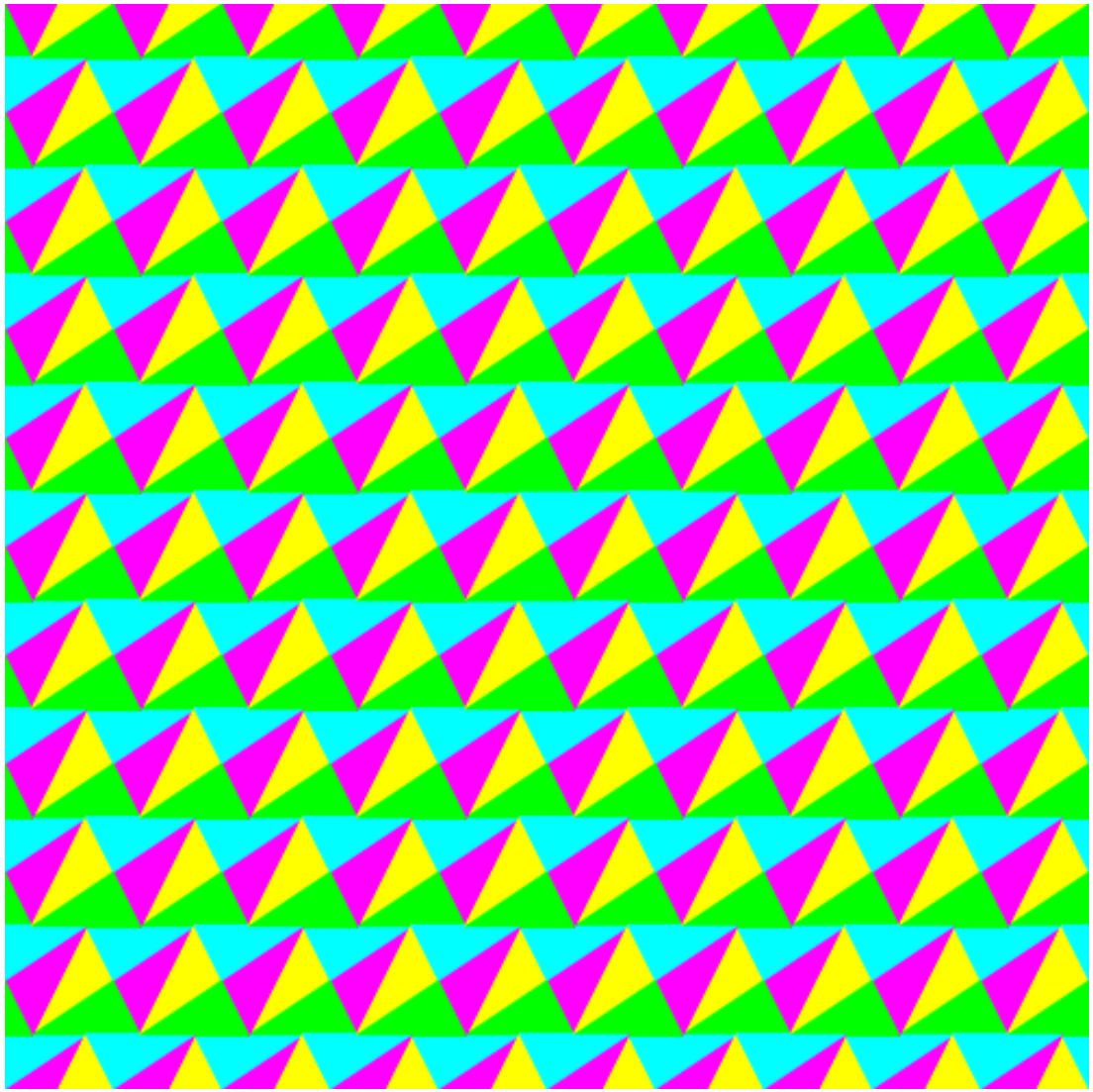


Figure: P1 wallpaper group

P4mm

- 4 rotation
- m is mirror

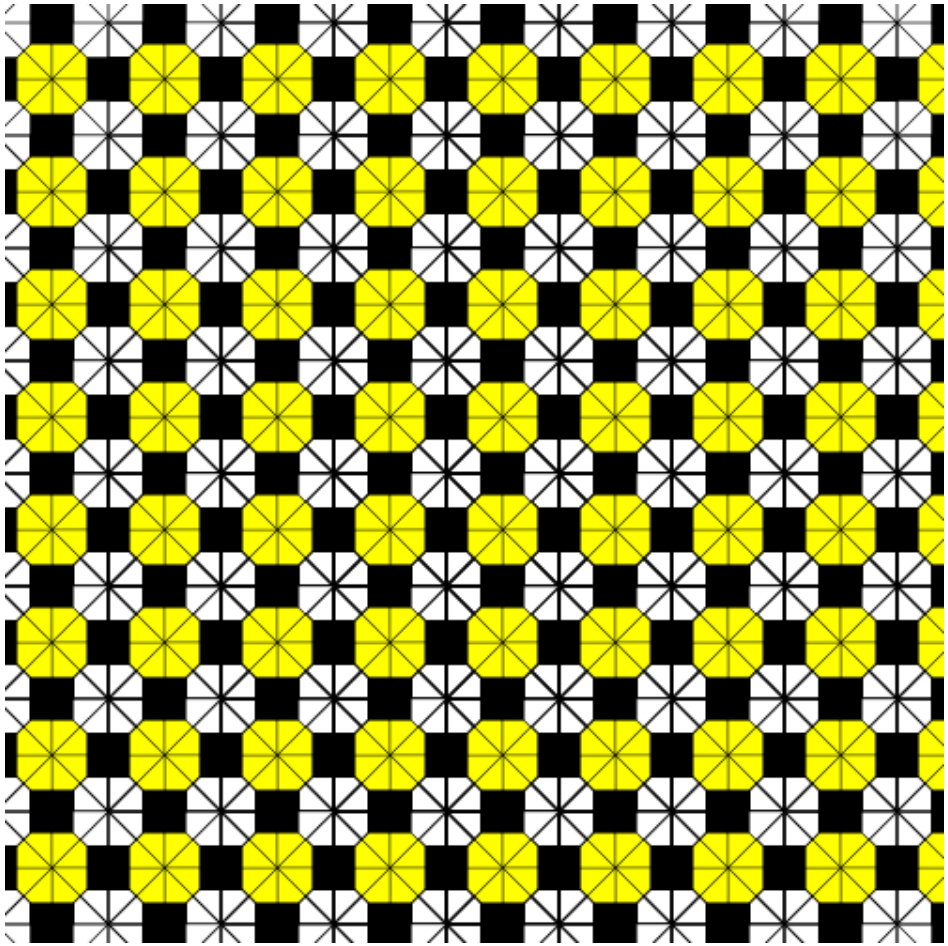


Figure: P4mm wallpaper group