Fields of definition of building blocks

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The building blocks B are the absolutely simple factors of the abelian varieties $J_1(N)$, the jacobians of the modular curves $X_1(N)$: they are abelian varieties defined over $\overline{\mathbb{Q}}$ only up to isogeny. Their endomorphism algebras belong to one of the following three types

CM: an imaginary quadratic field,

RM: a totally real field, or

QM: a quaternion algebra over a totally real field.

In this talk we describe a method to compute the endomorphism algebra of a building block B_f that is a factor up to isogeny of the Q-simple variety A_f attached by Shimura to a newform f, and to determine the number fields over which some representative in the isogeny class of B_f can be defined. The method can be used for practical computations and has been implemented in Magma. Some interesting examples and statistics will be given.