Problem Set 1 (due Friday, April 6th)

FROM THE TEXT:

Pages 267-268: 2 a, e, f, h, 3 a, b, c, d, 5

ADDITIONAL PROBLEMS:

A: Prove that $\mathbb{Q}(\sqrt{3}) \neq \mathbb{Q}(\sqrt{5})$.

B: Prove that $\mathbb{Q}(\sqrt{3},\ \sqrt{5}) \ = \ \mathbb{Q}(\beta)$, where $\beta = \sqrt{3} + \sqrt{5}$.

C: Let $p = 11213$ (which happens to be a prime number). Let $a = 571 + p\mathbb{Z}$, a nonzero element in the field $F = \mathbb{Z}/p\mathbb{Z}$. Find the additive and multiplicative inverses of a in the field F. You should express your answers in the form $r + p\mathbb{Z}$, where $0 \leq r < p$.

D: Let $F = \mathbb{Q}(\theta)$, where $\theta = \sqrt[3]{2}$ is the unique cube root of 2 in $\mathbb{R}$. Find the multiplicative inverse of $f = 2 + 3\theta - \theta^2$ in F. Express your answer in the form $f^{-1} = a + b\theta + c\theta^2$, where $a, b, c \in \mathbb{Q}$.

E: Let φ be an automorphism of the field $\mathbb{Q}$. Prove that $\varphi(a) = a$ for all $a \in \mathbb{Q}$.

F: Is $\mathbb{Q}[x]/(x^4 + 4)$ a field? Justify your answer carefully.