Some Possible Midterm Questions

1: Suppose that $a,\ m \in \mathbf{Z}$ and $m \geq 1$. Assume that $a^9 \equiv 1 \pmod{m}$. Prove that $gcd(a, m) = 1$.

2: Suppose that a, b, c, and m are integers, that $m \geq 1$, and that $abc \equiv 1 \pmod{m}$. Prove that $gcd(a, m) = 1$.

3: Suppose that $e \in \mathbf{Z}$ and that $e \equiv -18 \pmod{11}$. What can you say (if anything) about the remainder that e gives when divided by 55?

4: Suppose that $a \in \mathbf{Z}$ and that $(a+2)(a+4) \equiv 0 \pmod{13}$. What can you say (if anything) about the remainder that a gives when divided by 13?

5: Suppose that $b \in \mathbf{Z}$ and that b^2 gives a remainder of 9 when divided by 103. Prove that b gives a remainder of either 3 or 100 when divided by 103.

6: Suppose that b is an integer and that b^2 gives a remainder of 2 when divided by 23. Prove that b gives a remainder of 5 or 18 when divided by 23.

7: Suppose that $u, v \in \mathbf{Z}$ and that $gcd(u, v) = 1$. As proved in class, we know that there exist integers a and b such that $au + bv = 1$. Based on this fact, show how to prove the following statement:

$$\textit{If } u, v, r \in \mathbf{Z}, \quad u | vr, \textit{ and } gcd(u, v) = 1, \textit{ then } u | r.$$

(Remark: This statement to be proved is one of the versions of Euclid's Lemma.)

8: Suppose that $a, b, c \in \mathbf{Z}$ and that $gcd(a, c) = gcd(b, c) = 1$.

TRUE OR FALSE: *There exist integers e and f such that $abe + c^2 f = 5$.*

9: What is the remainder when $2^{17}5^8 4^2 13^{11}$ is divided by 7?

10 Let $n = 5^{50} + 13^{17} + 3^{15}$. Prove that n gives a remainder of 26 when divided by 39.

(Note that $39 = 3 \cdot 13$.)