Problem Set 6 (due on Wednesday, May 23rd)

A: The question concerns the integer $n = 2^{64} + 1$.

(a) Prove that if p is a prime which divides n, then $p \equiv 1 \pmod{128}$.

(b) The smallest prime p satisfying $p \equiv 1 \pmod{128}$ is $p = 257$. Prove that 257 does not divide n. (Hint: Notice that $257 = 2^8 + 1$ and hence that $2^8 + 1 \equiv 0 \pmod{257}$.)

B: Are there positive integers n with the following two properties: The last three digits of n (in base 10) are 111 *and* n gives a remainder of 32 when divided by 49. What can you say about the number of such integers n in the interval $0 < n < 200,000$?

C: A certain integer c gives a remainder of 5 when divided by 15. What can you say about the remainder that c gives when divided by 91?

D: Find all integers x such that $x \equiv 3 \pmod 5$, $\quad x \equiv 2 \pmod 7$, and $x \equiv 8 \pmod 9$.

E: Find all solutions to the congruence $x^2 + 1 \equiv 0 \pmod{130}$.