PROBLEM SET 5 (due on Monday, May 14th)

A: Determine $ord_{1155}(2)$.

B: Let $n = 22^{23} - 1$. Prove that n is divisible by 3 and by 7, but not divisible by 9.

C: Let $n = 22^{23} - 1$. Suppose that p is a prime that divides n. Prove that either $p = 3$ or $p = 7$ or $p \equiv 1 \pmod{46}$.

D: Let $n = 22^{23} - 1$. Prove that 47 does not divide n.

E: Prove that there are infinitely many positive integers n with the following two properties: All of the digits of n in base 10 are 1's *and* n is divisible by 49.

F: This problem concerns Euler's φ-function (which is also sometimes called Euler's totient function). Compute

$$\varphi(15), \qquad \varphi(11), \qquad \varphi(2^{99}), \qquad \varphi(24) \quad .$$