

PROBLEM SET 4 (due on Monday, May 7th)

A: Find all solutions to the following linear Diophantine equations:

(a) $56x + 72y = 40$

(b) $221x + 91y = 116$

(c) $221x + 91y = 117$

(d) $17x + 29y = 5$

B: Find all the solutions $z \in \mathbf{Z}$ to the following congruences:

(a) $56z \equiv 40 \pmod{72}$

(b) $17z \equiv 5 \pmod{29}$

(c) $29z \equiv 5 \pmod{17}$.

C: Find all solutions to the equation $30x + 17y = 1701$, where x and y are positive integers.

D: A man has \$4.55 in change consisting entirely of dimes and quarters. What is the maximum number of coins that he can have? What is the minimum number of coins that he can have?

E: Compute the following quantities:

$$\text{ord}_5(2), \quad \text{ord}_7(-2), \quad \text{ord}_{23}(2), \quad \text{ord}_{23}(5), \quad \text{ord}_{35}(3) .$$

F: Let $p = 11213$. It turns out that p is a prime. You may use that fact in this question. Find all integers b with the property that $\text{ord}_p(b) = 2$. Find all integers c such that $\text{ord}_p(c) = 3$.